Endogenous Firm Entry and the Supply-Side Effects of Monetary Policy*

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Abstract

This paper presents a model of the business cycle that highlights the importance of endogenous firm entry. In our framework, short-term supply shifts driven by new firm entries become a crucial factor in driving the economy's response to shocks, regardless of whether those shocks originate from the 'supply' or 'demand' blocks. Specifically, an uptick in aggregate demand triggers a cycle of increased firm entry, thereby enhancing aggregate supply and, in turn, further boosting demand through greater equipment purchases by new entrants. Monetary policy becomes especially powerful in this context, as it simultaneously impacts aggregate demand and the entry decisions of firms. This effect is particularly noticeable in economies with a significant potential for new firm entries. Our analytical approach characterizes the equilibrium of firm entry as a function of the 'policy room', a sufficient statistic related to monetary policy, which turns out to be positively correlated with the effectiveness of monetary and fiscal policy interventions both in the model and the data.

Keywords: Monetary Policy, Policy Multipliers, Endogenous Firm Entry
JEL Codes: D25, E32, E52

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1 Introduction

Contemporary macroeconomic models frequently classify individual shocks into separate 'demand' and 'supply' blocks.[1] However, distinguishing between them proves challenging in practice, as shocks often appear to intermingle and co-occur, as observed during the recent Covid-19 crisis. In this paper, we operationalize the concept of demand and supply separability (or lack thereof) in a precise manner, employing modern macroeconomic tools within a New Keynesian framework, and study the supply-side effect of monetary policy via endogenous firm entry and exit.

Figure 1 illustrates our basic problem via the classic aggregate demand (AD) and aggregate supply (AS) diagram. In the context of endogenous firm entry, a positive demand shock (AD_0 to AD_1) encourages additional supply via firm entry, as firms find it more profitable to enter the market. It shifts the aggregate supply curve from AS_0 to AS_1. As new entrants need to purchase necessary equipment for operating in the market, this shift in supply further amplifies aggregate demand (AD_1 to AD_2), initiating a self-reinforcing cycle between the two. Thus, firm entry generates the following three features: (i) a higher participation rate of firms mitigate the inflationary pressure and raise the output; (ii) demand and supply can be generally intertwined rather than separate entities, and shocks traditionally attributed to distinct demand-supply blocks have the potential to induce observationally similar co-movements in output and price; (iii) finally, the potency of monetary policy transmission on output will be stronger if endogenous responses in firm entry, thereby shifts in aggregate supply, are bigger.

In this context, the absence of demand-supply separability stems from the simultaneous co-movement of supply and demand, attributed to endogenous firm entry. This differs from the conventional equilibrium, which implies movement *along* the aggregate supply curve, rather than a shift of the curve itself when the economy faces demand shocks. To illustrate the importance of endogenous firm entry in explaining the business cycle, we offer a detailed analytical breakdown of labor adjustments in response to economic shocks, focusing on two key aspects: the extensive margin, which involves hiring by new entrants, and the intensive margin, which involves hiring by existing firms. Our analysis demonstrates that adjustments on the extensive margin are quantitatively significant in driving the economy's

[1] In a traditional macroeconomic framework, positive supply shocks (such as technology advancements or decreased cost-push factors) expand the supply curve and lead to lower equilibrium prices and increased production (as captured by the New Keynesian Phillips curve), while demand shocks generate a positive correlation between prices and production.

responses to various economic shocks.

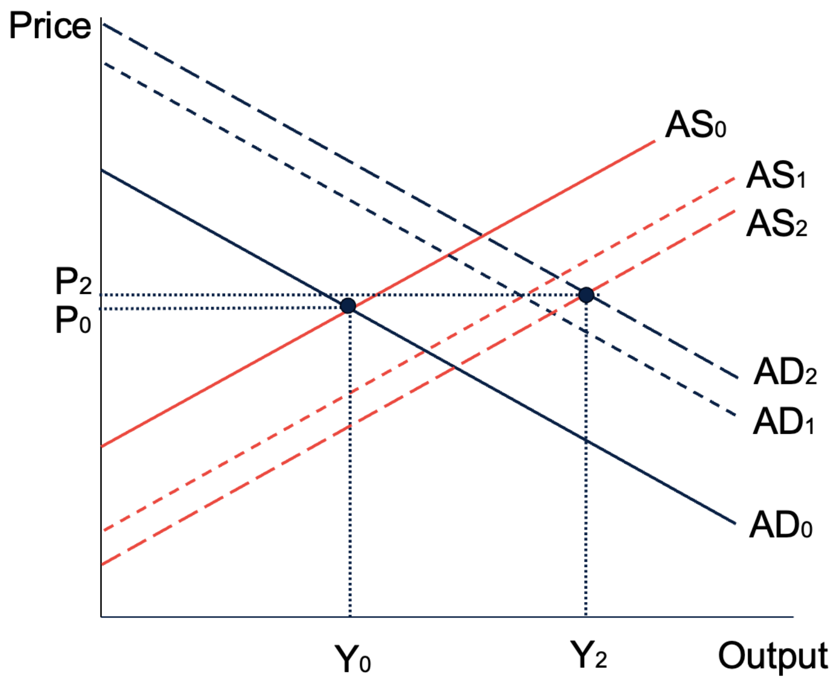

Figure 1: Convoluted aggregate demand and supply with endogenous firm entry

To facilitate the analysis, we decouple endogenous firm entry from other elements of the standard New Keynesian model by separating the production process across downstream and upstream industries. At the downstream level, a fixed measure of identical but differentiated firms engage in the production of a continuum of consumption varieties, face nominal pricing rigidities, and rely on upstream industry inputs. Upstream firms, conversely, enjoy price flexibility and employ labor, feature heterogeneous productivity endowments, and are obligated to incur stochastic fixed costs to enter the market and remain operational in subsequent periods. To further simplify the problem and obtain intuitive analytical expressions, we follow the literature on endogenous entry and assume that productivity and entry costs are drawn from independent Pareto distributions.[2] Finally, we impose a cash-in-advance constraint that, coupled with entry costs, generates upstream industry's reliance on borrowing from capital markets, connecting entry decisions to monetary policy via loan rates.[3] Therefore, monetary accommodation has dual roles: it raises aggregate demand as well as

[2]For the use of Pareto distributions for tractablity purposes, see e.g., Melitz (2003).

[3]Therefore, given a fixed cost level, a lower policy rate raises the likelihood that a firm operates in the market in the subsequent period.

encourages additional firm entries by reducing the loan rates faced by firms, triggering a self-reinforcing cycle between demand and supply.

Our model yields several interesting analytical outcomes, one of which is the formulation of a minimum policy rate, termed the "Satiation Bound (SB)", which is defined as the threshold rate that ensures full market participation of firms with comparable fixed costs. When the policy rate falls below SB, firms with even the lowest productivity will find market entry profitable. As a result,[4] market entry becomes less responsive to further monetary policy easing and other expansionary economic shocks. In such scenarios, the horizontal shift of the aggregate supply (AS) curve depicted in Figure 1 gradually diminishes as the policy rate falls. Consequently, the effectiveness of monetary policy in stimulating production diminishes, leading to reduced output multipliers and more pronounced inflationary responses, among others.

This observation suggests that the gap between the current policy rate and the average Satiation Bound (SB), which we refer to as the "policy room", acts as a *sufficient statistic* for gauging the supply-side impact of monetary policy. In fact, our model generates significant correlation between our "policy room" measure and the potency of monetary policy, as well as general responses to other shocks.

We then test the key model implication that the potency of monetary policy is strengthened as our "policy room" measure is higher. By presenting novel ways to recover the policy room, which is unobservable, we find that a higher (retrieved) policy room significantly increases the potency of monetary policy shocks: for example, one standard deviation increase (2 percentage points) in log-policy room raises a magnitude of output decrease (in %) in response to a one standard deviation tightening shock by around 3 percentage points, thus confirming our mechanism in the data.

Related literature Our business cycle setting with endogenous (upstream) firm entry follows previous works in the literature, e.g., Bilbiie et al. (2007), Bergin and Corsetti (2008),[5] Stebunovs (2008), Kobayashi (2011) Bilbiie et al. (2012), Uusküla (2016), Hamano and Zanetti (2017). While some papers assume equity financing for newly entering firms, e.g.,

[4]As firms with the lowest productivity have already entered the market, additional easing of monetary policy does not trigger a new wave of firm entry.

[5]Our assumption that fixed costs for market entry are paid in units of the final consumption goods aligns with the framework proposed by Bergin and Corsetti (2008). However, we deviate from their assumption of "pre-set" output procurement prices in favor of market prices.

Bilbiie et al. (2007), Bergin and Corsetti (2008), Bilbiie et al. (2012),[6] new firms finance their entry costs via borrowing from a loan market in our model, as in Stebunovs (2008), Kobayashi (2011), Uusküla (2016), so that firm entry is boosted under monetary accommodation, which aligns with the evidence presented in Colciago and Silvestrini (2022).[7] In addition, we express the equilibrium firm entry as a sole function of the "policy room", a sufficient statistic we devise. Up to our knowledge, we are one of the first works that devise the *sufficient statistic* that accounts for the supply effect of demand shocks and empirically test the channel.[8]

Our characterization of the Satiation Bound (SB) hinges on the idea that (i) monetary expansion facilitates an upswing in firm entry, and (ii) upon the monetary policy rate reaching a specified lower bound, all potential firms associated with a particular fixed entry cost have ventured into the market. Beyond this juncture, the positive supply effects stemming from further monetary accommodation and subsequent firm entry begin to wane. This phenomenon resonates with the insights of Ulate (2021) and Abadi et al. (2022), who incorporate analogous concepts in the context of banking profitability and the negative interest rates.

Layout Section 2 presents our New Keynesian model with endogenous firm entry. Section 3 discusses our calibration, steady-state analysis, and comparative statics. The model economy's impulse response functions to various shocks are explored in Section 4. Section 5 provides our empirical analysis and confirms the model predictions in the data. Concluding remarks are presented in Section 6.

Derivations and proofs are detailed in Appendix A. Appendix B summarizes the equilibrium conditions, inclusive of the flexible-price and steady-state benchmarks. Appendix C provides estimation techniques of the unobservable policy room and the satiation measure based on available data. For supplementary tables and figures, readers are directed to

[6]Under the equity financing for new entrants, an expansionary monetary shock leads to an increase in the aggregate demand for products, raising labor demand and wages. Higher labor costs for potential entrants can lower their net present value and reduce the entry rate of new firms, which is counterfactual. For the role of "real wage rigidity" in resolving this problem, see e.g., Lewis and Poilly (2012).

[7]Colciago and Silvestrini (2022) find the empirical evidence that expansionary monetary policy leads to an initial decrease and then an overshooting in the average productivity of the economy, as well as an initial increase and then undershooting in the firm's entry rate.

[8]Bergin and Corsetti (2008) similary find that monetary policy has a significant impact on the creation of new firms. Jordà et al. (2024) find long-run effects of monetary policy, through supply channels including capital stocks and the total factor productivity (TFP), while we focus on short-run supply effects of monetary policy.

Appendix D. Appendix E presents various robustness checks to our empirical analysis, and lastly, Appendix F provides the derivation of the model under a simplified framework with homogeneous entry costs.

2 Model

2.1 Representative Household

The representative household maximizes lifetime utility given by

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[\phi_{c,t} \cdot \log\left(C_t\right) - \left(\frac{\eta}{\eta+1}\right) \cdot N_t^{\left(\frac{\eta+1}{\eta}\right)} \right],$$

where C_t is consumption, N_t is labor, and $\phi_{c,t} \equiv \exp\left(u_{c,t}\right)$ is an aggregate demand shock defined as $u_{c,t} = \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right)$. The household's budget constraint is

$$C_t + \frac{D_t}{P_t} + \frac{B_t}{P_t} = \frac{R_{t-1}^D D_{t-1}}{P_t} + \frac{R_{t-1}^B B_{t-1}}{P_t} + \frac{W_t N_t}{P_t} + \frac{\Upsilon_t}{P_t},$$

where D_t represents bank deposits, and B_t denotes government bonds, which are in zero net supply in equilibrium. The corresponding gross interest rates for these assets are represented by R_t^D and R_t^B, respectively.[9] Υ_t captures lump-sum transfers to households. Such transfers may originate from various sources, including fiscal policies (such as subsidies to firms) or residual firm profits.

The first-order conditions bring the following standard intertemporal and intratemporal equations: The first-order conditions of this problem are

$$\frac{1}{R_t^D} = \frac{1}{R_t^B} = \beta E_t \left[\frac{\phi_{c,t+1}}{\phi_{c,t}} \cdot \frac{C_t}{C_{t+1}\Pi_{t+1}} \right], \tag{1}$$

$$N_t^{\frac{1}{\eta}} = \phi_{c,t} \cdot C_t^{-1} \cdot \frac{W_t}{P_t}. \tag{2}$$

The household is indifferent between investing in bonds or deposits in equilibrium, and central bank policy via R_t^B has a one-to-one pass-through on R_t^D.

[9] We do not consider issues pertaining to the zero lower bound (ZLB) in this paper, so it is possible for interest rates to be negative, $R_t^D < 1$.

2.2 Firms

The model stratifies firms into two discrete categories: those belonging to the downstream industry and those in the upstream industry. In both layers, firms operate in an environment of monopolistic competition. Notably, only downstream firms encounter nominal price rigidities à la Calvo (1983). Operational dynamics are structured such that upstream firms employ labor to generate intermediate input varieties, whose aggregator the downstream firms subsequently incorporate into the production of consumption good varieties. Representative households own firms across both industries, and consume the aggregated downstream goods.

One of the defining elements of our framework is the decision-making process for upstream firms. At each period, firms evaluate whether to continue/start operations in the next period. Should they decide to remain/enter the market, they must incur certain fixed costs, denominated in final goods, which are financed through loans from the banking sector.[10]

2.2.1 Downstream Industry: Aggregator

A representative firm, operating under perfect competition, aggregates the differentiated products produced by a continuum of downstream firms, denoted by u, spanning the interval $[0,\ 1]$. This can be formally expressed as:

$$Y_t = \left[\int_0^1 Y_t(u)^{\frac{\gamma-1}{\gamma}} \, \mathrm{d}u \right]^{\frac{\gamma}{\gamma-1}} .$$

The demand for each distinct variety produced by downstream firms, as well as the aggregate price, are given by

$$Y_t(u) = \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} Y_t \,, \tag{3}$$

$$P_t = \left[\int_0^1 P_t(u)^{1-\gamma} \, \mathrm{d}u \right]^{\frac{1}{1-\gamma}} ,$$

where $Y_t(u)$ and $P_t(u)$ are the output and prices of downstream varieties, respectively. Let $X_t = P_t Y_t$ represent the nominal aggregate expenditure, and $X_t(u) = P_t(u) Y_t(u)$ denote the expenditure for a specific downstream variety u. Given these definitions, the individual

[10]This dependency on external funding effectively functions as a cash-in-advance production constraint.

demands can be reformulated as:

$$X_t(u) = \Gamma_t \cdot P_t(u)^{1-\gamma} \ , \qquad \text{where: } \Gamma_t = X_t P_t^{\gamma-1}.$$

2.2.2 Downstream Industry: Monopolistic Competition with Sticky Prices

Consider a firm u within the downstream industry, belonging to the interval $[0,1]$. This firm employs $J_t(u)$ units of the aggregate product from the upstream industry and produces $Y_t(u) = J_t(u)$, indicating a one-to-one transformation from input to output. Consequently, the aggregate sum of upstream products, denoted as J_t, satisfies: $J_t \equiv \int_0^1 J_t(u)\ \mathrm{d}u = \int_0^1 Y_t(u)\ \mathrm{d}u$.

The profit equation for a downstream firm u is given by

$$\Pi_t(u) = (1+\zeta^T)P_t(u)Y_t(u) - P_t^J J_t(u) \ ,$$

where P_t^J represents the price of the aggregate upstream product, and ζ^T stands for a production subsidy to downstream firms. Thus, the present discounted value of profits, which the downstream firm u seeks to maximize, can be expressed as:

$$\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \left[(1+\zeta^T) P_{t+l}(u) Y_{t+l}(u) - P_{t+l}^J J_{t+l}(u) \right] \right\} \ ,$$

with $Q_{t,t+l}$ being the stochastic discount factor between time t and $t+l$.

Firms in the downstream industry face price stickiness à la Calvo (1983), characterized by a price-resetting probability of $1-\theta$. A firm, when adjusting its price P_t^*, aims to:

$$\max_{P_t^*} \sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left[(1+\zeta^T) P_t^* - P_{t+l}^J \right] \left(\frac{P_t^*}{P_{t+l}} \right)^{-\gamma} Y_{t+l} \right\} \ ,$$

where all firms that adjust their prices select P_t^* as the revised price. The resulting first-order condition can be articulated as:

$$\frac{P_t^*}{P_t} = \frac{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1} \right) \left(\frac{P_{t+l}}{P_t} \right)^{\gamma+1} \left(\frac{P_{t+l}^J}{P_{t+l}} \right) Y_{t+l} \right\}}{\sum_{l=0}^{\infty} E_t \left\{ Q_{t,t+l} \theta^l \left(\frac{P_{t+l}}{P_t} \right)^{\gamma} Y_{t+l} \right\}} \ . \tag{4}$$

2.2.3 Upstream Industry: Aggregator

There exists a continuum of upstream firms spanning the interval $[0, 1]$, each producing a distinct variety. These firms exhibit heterogeneity in two principal dimensions: productivity, indexed by v, and operational fixed costs, indexed by m. The output of a firm, uniquely identified by the index pair mv, is defined as $J_{mv,t}$. A perfectly competitive firm aggregates these upstream varieties as:

$$J_t = \left[\int_0^1 \int_{v \in \Omega_{m,t}} J_{mv,t}^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}v \, \mathrm{d}m \right]^{\frac{\sigma}{\sigma-1}} ,$$

where $\Omega_{m,t}$ denotes the subset of upstream firms sharing the same operational fixed cost m that decide to produce in period t. Given significant fixed costs, only the firms with the highest productivity levels may find production viable. The demand for an individual upstream variety (m, v), is:

$$J_{mv,t} = \left(\frac{P_{mv,t}^J}{P_t^J} \right)^{-\sigma} J_t \, . \tag{5}$$

Subsequently, the aggregate price index for the upstream product is:

$$P_t^J = \left[\int_0^1 \underbrace{\int_{v \in \Omega_{m,t}}^1 \left(P_{mv,t}^J \right)^{1-\sigma} \, \mathrm{d}v}_{\equiv (P_{m,t})^{1-\sigma}} \, \mathrm{d}m \right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 \left(P_{m,t}^J \right)^{1-\sigma} \, \mathrm{d}m \right]^{\frac{1}{1-\sigma}} , \tag{6}$$

where $P_{m,t}^J$ serves as the aggregate price of input for firms bearing the fixed costs indexed by m. We further define the nominal expenditure on a given upstream variety as $X_{mv,t}^J = P_{mv,t}^J J_{mv,t}$, and the aggregate expenditure as $X_t^J = P_t^J J_t$, so

$$X_{mv,t}^J = \Gamma_t^J \cdot P_{mv,t}^{1-\sigma} , \qquad \text{where: } \Gamma_t^J = X_t^J \left(P_t^J \right)^{\sigma-1} . \tag{7}$$

Using equation (3), we can express the aggregate input demand of downstream firms as:

$$J_t = \int_0^1 Y_t(u) \, \mathrm{d}u = Y_t \underbrace{\int_0^1 \left(\frac{P_t(u)}{P_t} \right)^{-\gamma} \mathrm{d}u}_{\equiv \Delta_t} = Y_t \Delta_t \, , \tag{8}$$

where

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1} \ , \tag{9}$$

represents a measure of price dispersion. Utilizing equation (8), equation (7) can be expressed as $\Gamma_t^J = (P_t^J)^\sigma Y_t \Delta_t$.

2.2.4 Upstream Industry: Monopolistic Competition, Loans, and Entry Decisions

The production function for an arbitrary firm (m, v) features diminishing returns to scale and is given by

$$J_{mv,t} = \varphi_{mv,t} \cdot N_{mv,t}^{\alpha}, \qquad \text{with } 0 < \alpha \leq 1 \ ,$$

where $N_{mv,t}$ denotes the labor employed, and $\varphi_{mv,t}$ is a firm-specific productivity assumed to be drawn from a Pareto distribution, $\varphi_{mv,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\kappa-1}{\kappa}\right)A_t, \kappa\right)$, with A_t being the average aggregate productivity. A higher κ implies that the productivity distribution is more concentrated around its mean, A_t. The cumulative distribution function is given by:

$$\Psi(\varphi_{mv,t}) = 1 - \left(\frac{\left(\frac{\kappa-1}{\kappa}\right)A_t}{\varphi_{mv,t}}\right)^{\kappa} \ ,$$

with the probability distribution function defined as $\psi(\varphi_{mv,t}) \equiv \Psi'(\varphi_{mv,t})$.

Profit Function: Firms must pay a pre-determined in-kind fixed cost, $F_{m,t-1}$, in the preceding period (i.e., at $t-1$) to operate in each period t. This cost, which covers expenses such as equipment acquisition, is assumed to be financed through loans financed at the prevailing gross rate, R_{t-1}^J. The profit for an upstream firm, if it chooses to operate in period t, is:

$$\Pi_{mv,t}^J = \underbrace{\left(1+\zeta^J\right)P_{mv,t}^J J_{mv,t}}_{\equiv r_{mv,t}} - W_t N_{mv,t} - R_{t-1}^J P_{t-1} F_{m,t-1} \ , \tag{10}$$

where ζ^J is a production subsidy to upstream firms and $r_{mv,t}$ represents their revenue. These upstream firms operate in a monopolistically competitive market and are not subject to nominal rigidities, setting prices as a constant markup over marginal costs (if they decide to produce), formally:

$$P_{mv,t}^J = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} \ . \tag{11}$$

By substituting the derived price equation into equation (10) and using the demand equations (5) and (7), we can rewrite the profit function as:

$$\Pi^J_{mv,t} = \Xi_t \cdot \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} \ , \tag{12}$$

where

$$\Xi_t \equiv \frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\frac{-\sigma}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \ . \tag{13}$$

Entry Decision: Firms' entry decision is taken one-period ahead in $t-1$, and is based on their expected profits and associated costs in t. We assume that firms know at $t-1$ their forthcoming productivity for period t, $\varphi_{mv,t}$. However, they remain uninformed about other eventual shocks that could impact individual demand in t.[11] Should a firm decide to operate, it will subsequently hire labor in t from the spot market, realizing profits as described in equation (12). Given the productivity draws, we can pinpoint the productivity threshold, $\varphi^*_{m,t}$, below which a firm would expect zero profit. Firms with the same fixed cost, $F_{m,t-1}$, and their productivity draw below this threshold will opt out of market entry for period t. Using equation (12), the formal representation of $\varphi^*_{m,t}$ is:

$$E_{t-1}\left[\xi_t \cdot \Xi_t\right] \cdot \left(\varphi^*_{m,t}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} - R^J_{t-1} P_{t-1} F_{m,t-1} = 0 \ , \qquad \text{where: } \xi_t = \frac{Q_{t-1,t}}{E_{t-1}\left[Q_{t-1,t}\right]} \ . \tag{14}$$

It's important to note that this threshold, $\varphi^*_{m,t}$, is based on *ex-ante* expected profits. Once a firm (m, v) commits to market entry, unforeseen shocks could potentially push profits into negative figures. Considering the inherent lower limit on productivity, $\left(\frac{\kappa-1}{\kappa}\right) A_t$, the actual productivity threshold for entry becomes $\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}$.[12] The proportion of firms with a fixed cost $F_{m,t-1}$ that decide to operate in t is denoted as $M_{m,t}$ and is given by

$$M_{m,t} \equiv Prob\left(\varphi_{mv,t} \geq \varphi^*_{m,t}\right) = \min\left\{ \left(\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1} F_{m,t-1}} \right)^{\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}}, 1 \right\} \ , \tag{15}$$

[11] This contrasts with Burnside et al. (1993), where labor decisions precede the realization of shocks. In our model, the decision to enter the market precedes the realization of other demand shocks. For simplicity, we assume that firms possess perfect foresight regarding their next period's productivity.

[12] If $\varphi^*_{m,t}$ is below $\left(\frac{\kappa-1}{\kappa}\right) A_t$, then all firms categorized by fixed cost m will operate in t.

where we use (14) to substitute for $\varphi^*_{m,t}$ in the last expression. From this equation, we can derive the following proposition:

Proposition 1 *For upstream firms with a fixed cost of $F_{m,t-1}$, $\underline{M_{m,t}=1}$ when the policy rate R^J_{t-1} is below a threshold $R^{J,*}_{m,t-1}$ given by*

$$R^{J,*}_{m,t-1} \equiv \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{P_{t-1}F_{m,t-1}} \ . \tag{16}$$

We refer to this threshold, $R^{J,*}_{m,t-1}$, as the "satiation bound" (SB) for firms of fixed cost type m.

As the policy rate, R^J_{t-1}, falls, more firms with the fixed cost $F_{m,t-1}$ opt for market entry in t due to the reduced loan repayment costs. Upon the policy rate reaching the type-specific bound $R^{J,*}_{m,t-1}$, all firms sharing the fixed cost $F_{m,t-1}$ (or lower) decide to become operational in t, leading to a stagnation in market entry for firms of cost type m and below. This fixed cost type-specific lower bound on the policy rate, $R^{J,*}_{m,t-1}$, is hence termed the satiation bound (SB).

In addition to the conventional intertemporal substitution effect captured by the Euler equation (1), monetary policy wields influence over the market entry decisions of upstream firms. This, in turn, impacts the input market's prices and quantities, cascading onto the aggregate economy via downstream product markets. Upon the rate hitting the SB for firms with the fixed cost $F_{m,t-1}$, no supplementary entries occur, rendering the supply-side effect of monetary policy ineffectual for such firms.

Loan Demand: From equation (15), we derive the expression for the aggregate real loan demand of firms with a fixed cost type m:

$$\frac{L_{m,t-1}}{P_{t-1}} = M_{m,t} \cdot F_{m,t-1} \ . \tag{17}$$

Firms opting to operate in period t borrow an amount $L_{m,t-1}$ to acquire final goods equivalent to $F_{m,t-1}$. This acquisition of final goods connects the entry decisions of firms to the aggregate demand of the economy via the loan channel.

Fixed Cost Distribution: We assume that the fixed costs of upstream firms, $F_{m,t}$, are drawn from a Pareto distribution, $F_{m,t} \overset{\text{iid}}{\sim} \mathcal{P}\left(\left(\frac{\omega-1}{\omega}\right)F_t, \omega\right)$, where F_t represents the average fixed cost associated with running a business, and $\omega > 1$ is the parameter that determines the variance of the distribution. The associated cumulative distribution function is:

$$H(F_{m,t}) = 1 - \left(\frac{\left(\frac{\omega-1}{\omega}\right) F_t}{F_{m,t}} \right)^{\omega} , \tag{18}$$

and its probability distribution function is denoted by $h(F_{m,t}) \equiv H'(F_{m,t})$. From Proposition 1, we obtain the probability measure of fixed cost types $F_{m,t-1}$ that are fully satiated, that is, the share of all firms with fixed cost $F_{m,t-1}$ that have already entered the market by time t, thus resulting in $M_{m,t} = 1$. This leads us to the following proposition:

Proposition 2 *Given the distribution in equation (18), the probability that $M_{m,t} = 1$ is:*

$$Pr\left(R^J_{t-1} \leq R^{J,*}_{m,t-1}\right) = Pr\left(F_{m,t-1} \leq \underbrace{\frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R^J_{t-1} P_{t-1}}}_{\equiv F^*_{t-1}} \right) \equiv H\left(F^*_{t-1}\right) ,$$

where F^*_{t-1} is the fixed cost threshold as defined above. All firms with a fixed cost $F_{m,t-1}$ less than or equal to F^*_{t-1}, irrespective of their productivity values $\varphi_{mv,t}$, opt to produce in period t. We term F^*_{t-1} the "full satiation fixed cost threshold".

Proposition 2 can be interpreted as follows: If a firm's fixed cost, $F_{m,t-1}$, is sufficiently low —below the threshold F^*_{t-1}— then even a firm with the lowest productivity draw would still deem operations in period t as profitable. Consequently, all firms bearing that fixed cost, regardless of their respective productivity draws, are active in period t.

Upstream Industry: Aggregation: The price aggregator for operating upstream firms, denoted by P^J_t, can be expressed as:

$$\frac{P^J_t}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F^*_{t-1}\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} , \tag{19}$$

where $\Theta_3 = \frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1 \omega(\sigma-1)}$ and $\Theta_4 = \frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}$ are constants. The aggregate measure of firms that operate during period t, represented by M_t, is given by

$$M_t = \int_0^1 \int_{v \in \Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right] , \tag{20}$$

where $\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)}$. Subsequently, the aggregate loan demand from operational upstream firms can be derived as:

$$\frac{L_{t-1}}{P_{t-1}} = \frac{1}{P_{t-1}} \int_0^1 L_{m,t-1} \, \mathrm{d}m = F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] , \qquad (21)$$

where $\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}$ is another model constant.

In equation (20), notice that as the satiation measure $H\left(F^*_{t-1}\right)$ increases, the number of operational firms at time t also increases. From equation (21), the aggregate real loan demand of firms is proportional to the average fixed cost, F_{t-1}, and grows with the satiation rate $H\left(F^*_{t-1}\right)$. Finally, in equation (19), the relative price of inputs from upstream firms relates to the technology-adjusted real wage, $\frac{W_t}{P_t A_t}$, and the aggregate demand for inputs of downstream firms, $\frac{Y_t \Delta_t}{A_t}$. When participation from upstream firms increases, as indicated by $H(F^*_{t-1})$, this relative price decreases. This is due to more upstream varieties being available to downstream firms, leading to greater competition and lower input prices. Therefore, the entry of new firms can reduce marginal costs for downstream firms and mitigate inflationary pressures.

Average SB: We obtain the average satiation interest rate of the economy by integrating over equation (16), and denote it by $R^{J,*}_{t-1}$. This rate serves as a measure of the satiation propensity of upstream firms. When the prevailing policy rate R^J_{t-1} exceeds this average, a marginal reduction in R^J_{t-1} can induce an entry of upstream firms into the market. According to equation (19), this market entry can lower average input prices and subsequently mitigate inflation. It can also boost aggregate demand and increase the price level, as new entrants take out loans to meet fixed costs, thus enabling the acquisition of fixed equipment for the production of final goods.

Proposition 3 *The aggregate satiation bound (SB) is expressed as:*

$$R^{J,*}_{t-1} = \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{\infty} R^{J,*}_{m,t-1} \, dH\left(F_{m,t-1}\right) = \left(\frac{\omega^2}{\omega^2 - 1}\right) \cdot \frac{F^*_{t-1}}{F_{t-1}} \cdot R^J_{t-1} , \qquad (22)$$

where F^*_{t-1} is the threshold fixed cost relative to the average fixed cost F_{t-1} in the economy.

If the threshold fixed cost for satiation, F^*_{t-1}, surpasses the economy's average fixed cost F_{t-1}, it signals an elevated likelihood of satiation across diverse fixed cost categories. Consequently, that results in a high value of the average SB rate, $R^{J,*}_{t-1}$, relative to the policy rate, R^J_{t-1}. In such a situation, a minor ease in R^J_{t-1} may not substantially stimulate the entry of new upstream firms.

Limit case, $\omega \to \infty$: Under our calibration, the fixed cost distribution $H(F_{m,t})$ collapses to its mean value, F_t, thereby becoming degenerate. This results in a uniform fixed cost across all firms. The economy's state —whether fully satiated or not— is determined by the relative sizes of the policy rate R^J_{t-1} and the mean satiation bound, $R^{J,*}_{t-1}$. Specifically, if $R^J_{t-1} < R^{J,*}_{t-1}$, all upstream firms enter the market and commence production in t. This simplified version of the model yields analytically tractable equilibrium expressions. Additional insights into the equilibrium conditions for this scenario are provided in Appendix F.

2.3 Shock Processes

The average fixed cost F_t is modeled as follows:

$$F_t = \phi_f \cdot \bar{Y}_t \cdot \exp(u_{f,t}) = \phi_f \cdot \frac{Y}{A} \cdot A_t \cdot \exp(u_{f,t}) \ , \tag{23}$$

where $u_{f,t} = \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t}$ and $\varepsilon_{f,t}$ is normally distributed with mean 0 and variance σ_f^2. Here, $\frac{Y}{A}$ is the steady-state output level adjusted for technology, and $\bar{Y}_t = \frac{Y}{A} \cdot A_t$ represents the balanced-growth path output.[13]

For technological progress, the model adopts:

$$GA_t \equiv \frac{A_{t+1}}{A_t} = (1+\mu) \cdot \exp\{u_{a,t}\} \ ,$$

where $u_{a,t} = \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t}$, and $\varepsilon_{a,t}$ is normally distributed with mean 0 and variance σ_a^2.

Additionally, government expenditure G_t is formulated as:

$$G_t = \phi_g \cdot Y_t \cdot \exp(u_{g,t}) \ , \tag{24}$$

where $u_{g,t} = \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t}$, and $\varepsilon_{g,t}$ is normally distributed with mean 0 and variance σ_g^2. It is assumed that the government maintains fiscal balance, levying a lump-sum tax, i.e., $T_{g,t} = G_t$ on the representative household each period.[14]

[13] We assume that F_t scales with balanced-growth-path output $\bar{Y}_t$, not the contemporaneous output Y_t. In practice, this assumption has minimal quantitative impact.

[14] Considering a zero net supply of government bonds, the government's budget constraint is upheld.

2.4 Central Bank

We assume that the central bank follows a Taylor rule for interest rate determination. The formal representation of this rule is given by:

$$R_t^B = R_t^J = R^J \cdot \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\tau_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\tau_y} \cdot \exp\{\varepsilon_{r,t}\} \ ,$$

where $\varepsilon_{r,t}$ is a normally distributed idiosyncratic monetary policy shock with mean 0 and variance σ_r^2. The variable $\bar{Y}_t$ denotes the balanced-growth path output level, and $\bar{\Pi}$ indicates the steady-state trend inflation rate. Financial markets are competitive, and the rate that households face, i.e., R_t^B, equals R_t^J in equilibrium, the loan rate faced by upstream firms.

2.5 Aggregation

Here, we aggregate the equations presented in Section 2.2 to obtain the economy-wide conditions. Consider first the aggregate labor demand N_t, given by

$$N_t = \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \ , \tag{25}$$

where $H_{t-1} \equiv H(F_{t-1}^*)$ for simplicity, and

$$\Theta_N = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \ . \tag{26}$$

From equation (25), it becomes evident that aggregate labor demand, N_t, is positively correlated with the demand for upstream varieties, denoted by J_t. Conversely, the demand for labor decreases as the satiation measure, H_{t-1}, rises. An increase in H_{t-1} results in a higher aggregate measure of operating firms, M_t, as indicated in equation (20). This increase consequently stimulates employment through new entrants on the extensive margin. However, this surge in market entry also exerts downward pressure on the relative input price, $\frac{P_t^J}{P_t}$, and dampens the individual labor demand of existing firms, $N_{mv,t}$, due to intensified competition. In practice, the latter effect dominates and the reduction in labor demand at the intensive margin outweighs the increase at the extensive margin induced by new market entrants, provided that J_t is held constant.

The real wage, based on the household's intratemporal optimization condition in equation (2) and equation (25), is given by

$$\frac{W_t}{P_t A_t} = \Theta_N^{\frac{1}{\eta}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\eta\alpha}} (1 + \Theta_4 H_{t-1})^{-\frac{\alpha+\sigma(1-\alpha)}{\eta(\sigma-1)\alpha}} \cdot \exp\{-u_{c,t}\} \ . \tag{27}$$

Substituting equation (27) into equation (19) yields:

$$\frac{P_t^J}{P_t} = \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{C_t}{A_t}\right) \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1 + \Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(\sigma-1)\alpha}} \cdot \exp\{-u_{c,t}\} \ . \tag{28}$$

Analysis of equations (25), (27), and (28) confirms that, given fixed aggregate demand measures such as C_t and J_t, an increase in H_{t-1} results in a reduction of both individual and aggregate labor demand. Consequently, this drives down the equilibrium wage. Hence, an increase in the entry of upstream firms exerts a deflationary impact on the economy, signaling a positive shift in aggregate supply.

Market clearing: Market clearing in this economy is given by

$$C_t + \frac{L_t}{P_t} + G_t = Y_t \ , \tag{29}$$

which, in conjunction with equations (21), (23), and (24), can be reformulated as:

$$\frac{C_t}{Y_t} = 1 - \phi_g \cdot \exp\{u_{g,t}\} - \phi_f \cdot \left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1} \cdot \left[1 - \Theta_L \cdot [1 - H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\{u_{f,t}\} \ . \tag{30}$$

Notice that real loan demand is present on the left-hand side of equation (29). When upstream firms opt to operate in the next period, they secure loans from financial institutions and utilize them to pay for in-kind fixed costs in terms of the final consumption good. This raises aggregate demand, exerting an inflationary influence in the economy as shown in equations (27) and (28): in those equations, stronger aggregate demand translates to inflation.[15]

[15] A Keynesian-cross structure becomes evident in equation (29) when endogenous entry of upstream firms is considered. As Y_t expands, the measure of operating upstream firms, M_t, along with their loan demand, $\frac{L_t}{P_t}$, rises, thus generating successive increments in demand.

Consequently, the entry of upstream firms into the market has the dual effect of shifting both the aggregate supply and demand curves. Depending on the relative magnitudes of these shifts, market entry can exhibit either inflationary or deflationary tendencies. Section 4 will elaborate on the economy's short-run responses to demand and supply shocks within this framework, underscoring the inherent linkage between the two.

In Guerrieri et al. (2023), a sectoral supply shock —such as the closing of high-contact sectors due to Covid-19— is more likely to become Keynesian, triggering a more substantial shift in aggregate demand than in supply, especially in multi-sector economies with incomplete markets. While their focus is primarily on economies where the sector affected by the supply shock either complements or utilizes inputs from unaffected sectors, our dual-layered structure (comprising downstream and upstream industries) enables an exploration of the reciprocal impacts between supply and demand. Specifically, in our model, a supply shock to upstream firms causes shifts in aggregate demand via the labor market and loan demand. Conversely, a demand shock initiates shifts in the upstream supply curve, affecting downstream supply primarily through their impact on input prices, and thereby resulting in successive rounds of demand shifts.

Average SB and satiation: Upon substituting equation (A.24) in Appendix A into (22), we obtain an expression for the average SB rate:

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right) \cdot (1-H_t)^{-\frac{1}{\omega}} \cdot R_t^B \ . \tag{31}$$

This expression allows us to interpret the "policy room", denoted as $\frac{R_t^B}{R_t^{J,*}}$, as a decreasing function of the satiation measure H_t.

Corollary 1 re-expresses the policy room $\frac{R_t^B}{R_t^{J,*}}$ as a *sufficient statistic* for the aggregate participation rate of firms, M_{t+1}. Importantly, a wider policy room level (i.e., higher $\frac{R_t^B}{R_t^{J,*}}$) amplifies the impact of monetary easing on the entry of upstream firms.[16] This rests on the following straightforward logic: a relatively high current policy rate R_t^B compared to the average SB, $R_t^{J,*}$, increases the scope for additional firms to enter the market as the policy

[16]This is consistent with the concave and decreasing function M_{t+1} in relation to the policy rate, R_t^B, as seen in (33).

rate declines.[17] Note from equation (31) above that

$$\frac{R_t^B}{R_t^{J,*}} \leq \frac{\omega + 1}{\omega} \; . \tag{32}$$

Corollary 1 *The total measure of upstream firms opting to operate in period $t+1$ is:*

$$M_{t+1} = 1 - \Theta_M \cdot \left[\left(\frac{\omega}{\omega + 1} \right) \cdot \frac{R_t^B}{R_t^{J,*}} \right]^{\omega} \; , \tag{33}$$

and a decrease in the policy room $\frac{R_t^B}{R_t^{J,*}}$ generates a larger increment in M_{t+1} when starting from a higher initial policy room level.

Proof. Directly from equation (33), we find:

$$\frac{dM_{t+1}}{d\left(\frac{R_t^B}{R_t^{J,*}}\right)} = -\omega\Theta_M \left[\left(\frac{\omega}{\omega + 1} \right) \cdot \frac{R_t^B}{R_t^{J,*}} \right]^{\omega - 1} \cdot \frac{\omega}{\omega + 1} < 0 \; ,$$

whose absolute magnitude is increasing in the level of $\frac{R_t^B}{R_t^{J,*}}$, given $\omega > 1$. ■

Flexible Price Model: Under flexible prices, the price of consumption varieties produced by downstream firms exhibits a constant markup over the cost of upstream inputs. Mathematically, this relationship is expressed as:

$$\frac{P_t}{P_t^J} = \frac{(1 + \zeta^T)^{-1}\gamma}{\gamma - 1} \; . \tag{34}$$

This establishes that the flexible price equilibrium is money-neutral, signifying that the policy rate R^J exerts no influence on the real allocation of resources. Additional equilibrium conditions are provided in Appendix A.

2.6 Summary Equilibrium Conditions

For analytical tractability, balanced growth path-adjusted variables are denoted with a tilde, for example, $\tilde{Y}_t \equiv \frac{Y_t}{A_t}$. In our simulation results, we assume the government implements optimal transfers to neutralize real distortions arising from monopolistic competition. Specif-

[17] This pertains to scenarios where the fixed cost cutoff F_t^* is low, thus allowing middle-range fixed cost firms with suboptimal productivity to enter the market.

ically, this involves setting $\zeta^T = \frac{1}{\gamma-1}$ and $\zeta^J = \frac{1}{\sigma-1}$. A comprehensive list of equilibrium conditions is provided in Appendix B.

3 Steady State Results

3.1 Calibration and Estimation

The calibrated parameters are presented in Table 1. Our model incorporates two key factors influencing the operation of upstream firms in the market: fixed costs and productivity. These two variables follow their own independent Pareto distributions. The model is designed such that the proportion of operating upstream firms is sensitive to parameters associated with these Pareto distributions. Utilizing the calibrated parameters outlined in Table 1, our model effectively replicates the moments commonly targeted in the literature. Key steady-state values are displayed in Table 2.

Fixed Cost to Balanced-Growth-Path Output Ratio, ϕ_f: In Appendix C.2 and Appendix C.3 of Appendix C, we estimate $(\phi_f, \rho_f, \sigma_f)$ of equation (23), based on the available data on the number of establishments in the Quarterly Census of Employment and Wage (QCEW) database as a proxy for firm participation in the model.[18]

Our estimated $\phi_f = 0.5547$ yields 91% firm participation rate at the steady state (i.e., $M = 0.91$ in Table 2). It actually matches with the number based on exit and entry rates of establishments: according to the Business Dynamics Statistics (BDS), the average annual exit and entry rates from 1977 to 2016 were 10.6% and 12.3%, respectively. Our estimated value of $\phi_f = 0.5547$ yields a steady-state participation rate $M = 0.91$, in which the exit rate is around 10%.[19] Note that we do not target this number, as we estimate the entire fixed cost process (23) with available data.

Shape Parameters in Pareto Distributions, κ and ω: We select $\kappa = \omega = 3.4$ based on the work of Ghironi and Melitz (2005), who choose this shape parameter for the productivity distribution to align with the standard deviation of log U.S. plant sales, estimated

[18] Appendix C offers an alternative estimation method based on the total number of employees from CES National Databases in the Bureau of Labor Statistics (BLS) as well. See Appendix C.1.

[19] The fixed cost in our model is regarded as a composite of capital and non-capital costs. In the literature, the capital-to-output cost ratio is approximately estimated to be around 30%. According to Table 5 in Domowitz et al. (1988), the non-capital fixed cost-to-output ratio varies between 0.05 and 0.18 across industries. If we add up these two components, we see our estimated $\phi_f = 0.5547$ is at the upper end of this range.

at 1.67 by Bernard et al. (2003). In Appendix A.2, we provide a formula for our model-implied standard deviation of log revenues of upstream firms and compare with Bernard et al. (2003).

Elasticity of Substitution, γ and σ: We select $\gamma = 4.3$ for the elasticity of substitution in the downstream market, following Ghironi and Melitz (2005) where 30% mark-up of price over cost is documented.

We choose $\sigma = 3$ based on Jones (2011), who argue that the elasticity of substitution for upstream market products tend to be lower than in downstream markets. This number is also close to the number ($\sigma = 3.79$) used by Bernard et al. (2003), who calibrate the elasticity of substitution to align with U.S. plant-level and macro trade data. There, the value of $\sigma = 3.79$ is chosen to match the productivity and size advantages of U.S. exporters.[20] Our quantitative results turn out to be robust across different levels of σ around 3.

	Parameter Description	Value	Source
β	Discount factor	0.998	Dordal-i-Carreras et al. (2016).
η	Frisch labor supply elasticity	1	Standard.
γ	Elasticity of substitution (of downstream market)	4.3	From Ghironi and Melitz (2005): 30% mark-up of price over cost.
σ	Elasticity of substitution (of upstream market)	3	Lower elasticity of upstream market products as argued in Jones (2011).
α	labor share in the upstream production function	0.6	Standard.
θ	Calvo (1983) price stickiness	0.75	Standard.
κ	Shape parameter: Pareto distribution of productivity	3.4	Ghironi and Melitz (2005).
ω	Shape parameter: Pareto distribution of fixed cost	3.4	Keep it the same with the productivity distribution.
ϕ_f	Fixed cost - steady state output ratio	0.5547	Estimated (Appendix C.3)
ϕ_g	Government spending - output ratio	18%	Smets and Wouters (2007) (g_y).

[20] Several studies, including Ghironi and Melitz (2005), Bilbiie et al. (2012), and Fasani et al. (2023), also adopt the elasticity of substitution around this number, following Bernard et al. (2003).

τ_π	Taylor parameter (inflation)	1.5	Standard.
τ_y	Taylor parameter (output)	0.15	Standard.
μ	Long-run TFP growth rate	0.005	Match a yearly growth rate at 2%.
Π	Long-run inflation	1.02	Long-run inflation target at 2%.
ρ_a	Autoregression for TFP	0.7	0.85 in Schmitt-Grohé and Uribe (2007). Higher ρ_a makes our numerical solution unstable.
ρ_c	Autoregression for demand shock	0.98	The autocorrelation of the preference shock that affects the marginal utility of consumption estimated by Nakajima (2005).
ρ_g	Autoregression for government spending	0.87	Schmitt-Grohé and Uribe (2007).
ρ_f	Autoregression for fixed cost	0.9011	Estimated (Appendix C.3).
σ_a	SD for ϵ_a	0.0064	Schmitt-Grohé and Uribe (2007).
σ_c	SD for ϵ_c	0.017	The standard deviation of the preference shock estimated by Nakajima (2005) using U.S. data on consumption, labor, and output is 0.017.
σ_g	SD for ϵ_g	0.016	Schmitt-Grohé and Uribe (2007).
σ_f	SD for ϵ_f	0.0013	Estimated (Appendix C.3).
σ_r	SD for ϵ_r	0.0025	25 basis points, following Fed practices.

Table 1: Calibrated parameters.

Variable	Value	Description
H	0.82	Mass of productivity-irrelevant firms.
M	0.91	Mass of firms operating in the market.
R^B	1.012	Gross risk-free rate.
$R^{J,*}$	1.296	Gross satiation rate.
$\tilde{F}^*$	0.72	Cutoff fixed cost-to-output ratio.
Δ	1.0007	Price dispersion.
$\frac{W_t}{P_t A_t}$	0.51	Real wage.
$\frac{C_t}{Y_t}$	0.36	Consumption-to-output ratio.
$\frac{W_t N_t}{P_t Y_t}$	0.6	Labor cost-to-output ratio.
$\frac{L_t/P_t}{Y_t}$	0.46	Loan-to-output ratio.

Table 2: Steady state values.

3.2 Comparative Statics

In this section, we conduct comparative statics analyses on the steady-state equilibrium under varying parameter calibrations. This will illustrate the relationship between individual parameters and the internal mechanics of the model.

Fraction of Operating Upstream Firms: The steady-state proportion of active upstream firms, denoted as M, is described by $1 - \Theta_M[1 - H]$, as derived from equation (20). Figure 2 visualizes how M responds to shifts in model parameters: κ, ω, ϕ_f, β, μ, and Π. We

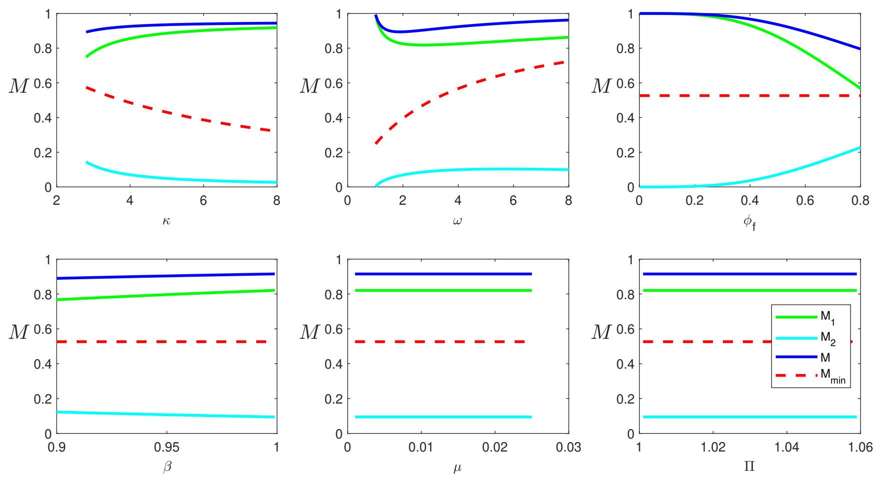

Figure 2: Comparative Statics: M.

Notes: Benchmark parameters are fixed as listed in Table 1. Ranges for κ, ω, ϕ_f, β, μ, and Π are $[2.8, 8]$, $[1.01, 8]$, $[0.001, 0.6]$, $[0.9, 0.999]$, $[0.001, 0.025]$, and $[1.001, 1.0709]$, respectively. The red dashed line marks the minimum mass of active firms, $M_{\text{min}} = 1 - \Theta_M$, attained when no firm is satiated, $H_t = 0$. We partition M into productivity-irrelevant M_1 and jointly determined M_2 components for various parameter values.

decompose M as follows:

$$M = Prob(F < F^*) + \cancel{Prob(F > F^*)} \int_{F^*}^{\infty} \left(\frac{F_m}{F^*}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{dH(F_m)}{\cancel{1 - H(F^*)}}$$

$$= \underbrace{H(F^*)}_{\equiv M_1} + \underbrace{\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + \omega(\sigma-1)}(1 - H(F^*))}_{\equiv M_2}.$$

Here, $M_1 = H(F^*)$ represents the mass of firms with sufficiently low fixed costs ($F_{m,t} \leq F^*$) to remain active irrespective of their productivity. M_2 comprises firms that are operational but not at the lowest fixed-cost tier; these firms do not operate if they draw a low productivity level.

The following key points can be drawn from Figure 2: (i) An increase in κ raises both M_1 and M by narrowing the productivity distribution around its mean, thereby raising the lower bound of productivity and the likelihood of satiation for any given fixed cost; (ii) An increase in ω manifests via two opposing effects on firm participation, M. On one hand, it raises the minimum fixed cost $\frac{\omega-1}{\omega}F$, thereby reducing M. On the other hand, it narrows the fixed-cost distribution around its mean F, potentially reducing the mass of high fixed-cost firms and increasing M. The total effect on M depends on the relative magnitudes of these two forces. Moreover, the satiation measure M_1 typically declines as ω increases due to an increased lower bound on fixed costs, $\frac{\omega-1}{\omega}F$, affecting firms that are typically satiated. These characteristics relating ω and M are further elaborated in Figure D.6 in Appendix D, which explores the effect of other parameters on the functional relationship between M and each parameter; (iii) An increment in ϕ_f shifts the fixed-cost distribution to the right, thereby reducing both M and M_1.

Following from equation (33), it is evident that the policy room $\frac{R^B}{R^{J,*}}$ maintains an inverse relationship with M. Variations in the parameters will produce effects on the policy room that are opposite to their impacts on M, as documented in Figure D.7 in Appendix D.

The Real Loan-to-Output Ratio: At the steady state, the following inequality is derived from equations (21) and (32):

$$\phi_f\,(1-\Theta_L) \leq \frac{L/P}{\bar{Y}} = \phi_f\left[1-\Theta_L(1-H(F^*))^{\frac{\omega-1}{\omega}}\right] = \phi_f\left[1-\Theta_L\left(\frac{\omega}{\omega+1}\frac{R^B}{R^{J,*}}\right)^{\omega-1}\right] \leq \phi_f\ ,$$

where the real loan-to-output ratio, $\frac{L/P}{\bar{Y}}$, is a decreasing function of the policy room $\frac{R^B}{R^{J,*}}$, but increasing with respect to the satiation measure $H(F^*)$, and total firm participation, M.[21]

Figure 3 describes how $\frac{L/P}{\bar{Y}}$ varies with key model parameters: κ, ω, ϕ_f, β, μ, and Π. Our observations can be summarized as follows: (i) An increase in κ raises firm participation M, as illustrated in Figure 2, and narrows the policy room $\frac{R^B}{R^{J,*}}$, as seen in equation (33) and Figure D.7, resulting on a higher aggregate loan demand; (ii) An increase in ω gives rise to conflicting outcomes: it initially depresses firm participation M when ω is below

[21]Note that M increases with H at the steady state as per equation (20).

a certain threshold, which can be attributed to an increase in the minimum fixed cost of entry, $\frac{\omega-1}{\omega}F$, as seen in Figure 2. However, this negative extensive margin effect is eventually counterbalanced by a positive intensive margin effect, where each active firm incurs a greater fixed cost, hence raising the real loan-to-output ratio; (iii) An increase in ϕ_f results in a reduction of firm participation M, evident from Figure 2, thus reducing aggregate loan demand. As before, this decrease via the extensive margin is eventually neutralized by an increase via the intensive margin, where each active firm shoulders a higher fixed cost.[22] The dynamics between the policy room $\frac{R^B}{R^{J,*}}$ and the real loan-to-output ratio $\frac{L/P}{Y}$ are cap-

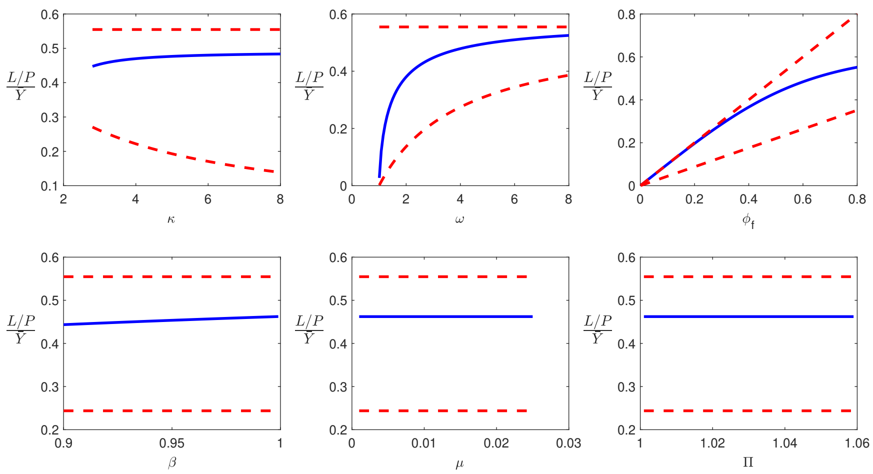

Figure 3: Comparative statistics: Output-scaled real lending.

Notes: The red-dashed lines indicate the upper and lower bounds for output-scaled lending, corresponding to ϕ_f and $\phi_f(1-\Theta_L)$, respectively.

tured in Figure 4. An increase in ϕ_f or ω lowers firm participation M and widens the policy room, $\frac{R^B}{R^{J,*}}$, with the net effect being an increase of aggregate loan issuance. In contrast, a rise in κ raises both M and $\frac{L/P}{Y}$, inducing a negative correlation with the policy room $\frac{R^B}{R^{J,*}}$.

[22]The functional relationship between $\frac{L/P}{Y}$ and other parameters is further explored in Figure D.8.

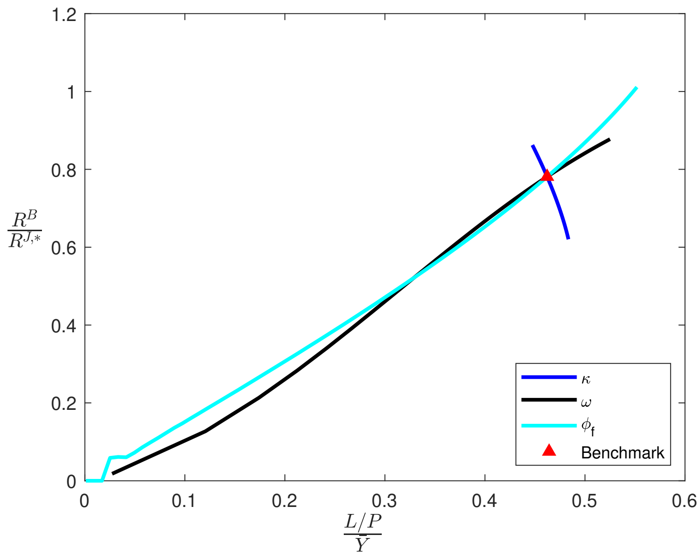

Figure 4: Policy power on output-scaled real lending.

Notes: This figure illustrates the co-movements between $\frac{R^B}{R^{J,*}}$ and $\frac{L/P}{\bar{Y}}$ driven by variations in κ, ω, and ϕ_f. The solid triangular marker denotes the steady-state value under benchmark calibration.

4 Quantitative Analysis

4.1 Supply vs. Demand Shocks

Technology Shock Figure 5 shows how a positive technology shock, $u_{a,t}$, affects various variables in our model. Following the shock, a group of previously inactive firms enters the market, boosting aggregate firm participation M_t, the measure of productivity-insensitive entrants H_t, and aggregate loans $\frac{L_t}{P_t A_t}$.[23] As participating firms pay their fixed costs in units of the final consumption good, the increase in firm entry contributes to an expansion in aggregate demand, as detailed in equation (29). An uptick in market participation typically depresses the real price of inputs, $\frac{P_t^J}{P_t}$, due to heightened competition, as expressed in equation (28). Yet in this case, the rising aggregate demand effect dominates, increasing real input prices along with labor demand N_t and real wages. This causes inflation Π_t and

[23] In Figures 5 and 6, the percentage increase in the loan-to-output ratio, $\frac{L_t/P_t}{\bar{Y}_t}$, is equal to $\frac{L_t}{P_t A_t}\frac{A}{Y}$, coming from a net rise in aggregate loan demand, $\frac{L_t}{P_t A_t}$. For small values of ϕ_f, changes in loan demand around the steady state are negligible.

interest rates R_t^B to rise, thereby narrowing the policy room $\frac{R_t^B}{R_t^{J,*}}$.[24]

We also examine the technology shock's impact under varying levels of the fixed cost parameter, ϕ_f. Higher entry costs mean a greater steady-state prevalence of inactive firms, $1-M$. In such conditions, a positive $u_{a,t}$ shock triggers substantial new firm entry and larger increases in M_t and H_t. The increase in aggregate demand brought by stronger entry is further amplified by the elevated fixed costs associated with a higher ϕ_f. Consequently, there's a sharper initial increase in loan demand, real input prices, wages, and labor demand, followed by a faster reversion to steady-state levels due to increased competition. In this setting, inflation Π_t shows a more moderate response due to larger shifts in firm entry.[25]

These dynamics align with the traditional AD-AS framework as follows: (i) a positive technology shock moves the supply curve rightward; (ii) it leads to an outward movement of the demand curve due to increased loan and labor demands, causing more firm entry and further shifts in the supply curve; and (iii) when entry costs are high, more inactive firms enter the market after a positive supply shock. Consequently, both the aggregate supply and aggregate demand curves shift more extensively rightward, resulting in moderate inflation and increased output.

Demand Shock Figure 6 illustrates the effects of a consumption demand shock, $u_{c,t}$. The figure exhibits impulse responses that are qualitatively analogous to Figure 5. Specifically, a positive shock to $u_{c,t}$ prompts an increase in firm entry that results in an expansion of the aggregate supply capacity of the economy.[26]

In summary, our model highlights the reciprocal relationship between supply and demand that exists as a result of endogenous firm entry. Accordingly, the initial origin of the shock —be it supply- or demand-driven— yields no qualitative distinctions in the behavior of the key variables within our model. Nonetheless, economies with a larger pool of potential new entrants generate stronger responses to shocks in the form of larger output and moderate inflation movements.

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[24]This result is consistent with the positive correlation between the policy room, $\frac{R_t^B}{R_t^{J,*}}$, and firm participation, M_t, outlined in equation (33)

[25]This observation is consistent with the findings of Cecioni (2010), who argue that greater firm entry can mitigate inflationary pressures in the U.S. economy.

[26]Note one difference between Figures 5 and 6: with our demand shocks, inflation drops due to stronger effects of additional firm entry on aggregate supply. Also, note that demand shocks are more persistent since $\rho_c >> \rho_a$ in our calibration.

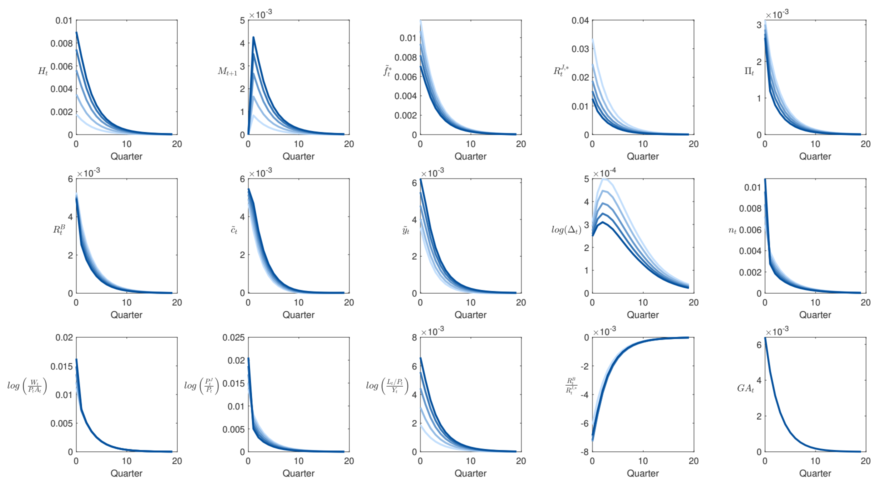


Figure 5: Impulse response functions to TFP shock.

Notes: The figures display the deviation for 1 standard deviation (0.01) in $u_{a,t}$ which increases the growth rate of the average productivity for upstream firms. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$. The rest of the variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ is the price dispersion for the downstream products.

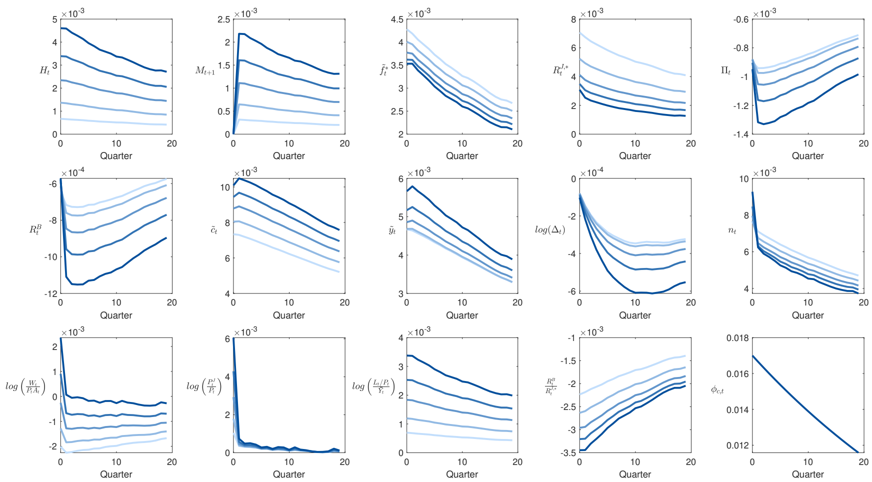


Figure 6: Impulse response functions to demand shock.

Notes: The figures display the deviation for 1 standard deviation (0.08) in $u_{c,t}$, the demand shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The below variables are plotted in deviations in level from their steady states: H, M, R^B, Π, and $R^{J,*}$. The rest of the variables are plotted in deviations in logs from their steady states (in lower case letters or with a log). Δ is the price dispersion for the downstream products.

Other Shocks In Appendix D.2, impulse response functions are presented for fixed cost shocks $u_{f,t}$ (Figure D.9), monetary policy shocks $\varepsilon_{r,t}$ (Figure D.10), and government spending shocks $u_{g,t}$ (Figure D.11). A positive fixed cost shock induces falls in firm entry M_t and the satiation measure H_t. This decline is attributed to the elevated productivity cutoff $\varphi^*_{m,t}$, as specified in equation (14), which rises for each firm type m due to increased entry costs. This shock has dual, opposing impacts on aggregate demand: First, reduced firm participation diminishes fixed equipment demand at the extensive margin, thereby contracting aggregate demand. Second, the increased fixed costs boost demand from incumbent firms, thereby augmenting aggregate demand at the intensive margin. Under the model's benchmark calibration (i.e., $\phi_f = 0.5547$), the latter effect prevails, leading to a net expansion in output. This subsequently results in an increase in equilibrium levels of labor demand, real wages, and inflation.

A positive monetary policy shock generates an impulse response function akin to that produced by a consumption demand shock. A rise in monetary policy rates lowers aggregate participation M_t, which in turn decrease loan demand, inflation, real wages, and production levels. A positive government spending shock, depicted in Figure D.11, crowds out consumption via higher real interest rates while simultaneously reducing inflation through increased participation by upstream firms, as evidenced by rises in both M_t and H_t. The government spending multiplier is amplified under higher values of ϕ_f, which is attributable to stronger firm entry following the shock.

4.2 Intensive vs. Extensive Margin in Labor Adjustment

Note that changes in aggregate labor N_t as specified in equation (25) are attributable to two primary factors: (i) variations in each operating firm's labor demand, denoted $N_{mv,t}$, over time —referred to as intensive margin adjustment; and (ii) fluctuations in the number of active upstream firms M_t across business cycles —known as extensive margin adjustment. The aggregate labor N_t is formally expressed in equation (35) as:

$$N_t = \int_0^1 \int_{v \in \Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m \,, \tag{35}$$

where the individual labor demand $N_{mv,t}$ derives from equation (A.15). We now proceed to consider an upstream firm (m, v) operating across two periods t and $t + \iota$, where $\iota \geq 1$.

Utilizing equation (A.15), we define:

$$g^{\text{Density}}_{t,t+\iota} \equiv \frac{N_{mv,t+i} - N_{mv,t}}{N_{mv,t}} = \left[\frac{1 + \Theta_4 \cdot H_{t-1}}{1 + \Theta_4 \cdot H_{t+\iota-1}}\right]^{\frac{\sigma}{(\sigma-1)\alpha}} \left(\frac{\frac{Y_{t+\iota}\Delta_{t+\iota}}{A_{t+\iota}}}{\frac{Y_t \Delta_t}{A_t}}\right)^{\frac{1}{\alpha}} - 1 \,, \tag{36}$$

which represents the percentage change between periods t and $t + \iota$ in an individual firm (m, v)'s labor demand $N_{mv,t}$, contingent upon the firm's operation in both periods. Importantly, $g^{\text{Density}}_{t,t+\iota}$ is solely a function of aggregate variables, independent of the indices (m, v). We term $g^{\text{Density}}_{t,t+\iota}$ as the "intensive margin" adjustment in labor demand.

From equation (25), we can derive an expression for the percentage change in aggregate labor, N_t, denoted as $g^N_{t,t+\iota}$[27]:

$$g^N_{t,t+\iota} \equiv \frac{N_{t+\iota} - N_t}{N_t} = g^{\text{Density}}_{t,t+\iota} + (1 + g^{\text{Density}}_{t,t+\iota}) \cdot g^{\text{Entry}}_{t,t+\iota} \,, \tag{37}$$

where $g^{\text{Density}}_{t,t+\iota}$ is defined as in equation (36) and $g^{\text{Entry}}_{t,t+\iota}$ is given by

$$g^{\text{Entry}}_{t,t+\iota} = \frac{(H_{t+\iota-1} - H_{t-1}) + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(H_{t-1} - H_{t+\iota-1})}{H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1 - H_{t-1})} \,. \tag{38}$$

We interpret $g^{\text{Entry}}_{t,t+\iota}$ as the extensive margin adjustment in labor, triggered by changes in firm entry. According to equation (37), the total percentage change in aggregate labor comprises both intensive and extensive margin adjustments.

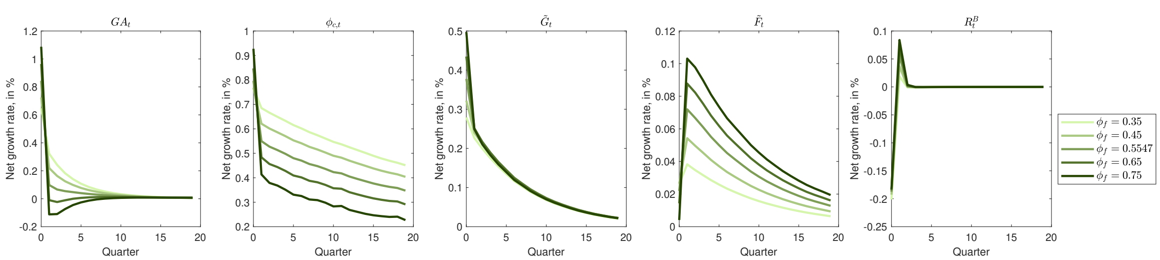


Figure 7: Decomposition of labor growth rate under different shocks: isolines on intensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient green lines indicate intensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

[27] The derivation is provided in Appendix A.

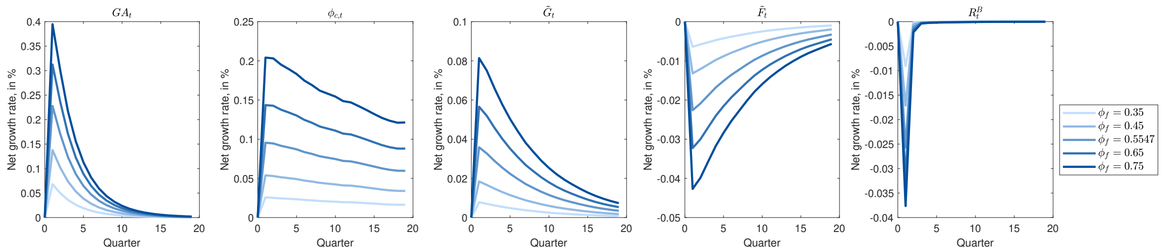


Figure 8: Decomposition of labor growth rate under different shocks: isolines on extensive margin.

Notes: Figures illustrate employment growth rate relative to pre-shock employment level. Gradient blue lines indicate extensive margin responses with varying fixed cost parameter ϕ_f values. Growth rates are reported in net percentage terms.

Figures 7 and 8 portray how intensive and extensive margins' elements respond, respectively, to different shocks. For example, for a positive fixed cost shock $u_{f,t}$, we note that (i) a negative extensive margin adjustment due to the exit of less competitive firms, and (ii) an increase in per-firm labor demand corresponding to higher aggregate output, as evidenced in Figure D.9.

In contrast, a consumption demand shock $\phi_{c,t}$ generates positive adjustments on both margins due to increased market entry and aggregate output (see Figure 6). The extensive margin effect becomes more salient under higher ϕ_f, while the intensive margin exhibits a non-monotonic behavior. Initially, individual firms require more workers, but as market competition intensifies, labor demand flattens, as corroborated by Figure D.9.

4.3 Multipliers and the Policy Room

We now examine the influence of "initial policy room levels" on the responses of aggregate variables to shocks, commonly termed in the literature as shock multipliers. To obtain the value of multipliers outside the steady state, we simulate the model over a span of $T = 10,000$ periods, selecting 500 unique realizations denoted as $\mathbb{Y}^{\text{original}}$. For each selected realization, we extend the model dynamics up to $h = 4$ periods ahead based on two different scenarios: (i) no additional shocks, which results in the time series $\left\{\mathbb{Y}^{\text{original}}_{t+h}\right\}_{h=0}^{h=4}$; and, (ii) an initial one standard deviation addition to the shock of interest, giving rise to the time series $\left\{\mathbb{Y}^{\text{shock}}_{t+h}\right\}_{h=0}^{h=4}$. The multiplier is subsequently computed as $\frac{|\mathbb{Y}^{\text{shock}}_{t+h} - \mathbb{Y}^{\text{original}}_{t+h}|}{\sigma(\text{shock})}$ for horizons ranging from $h = 0$ to $h = 4$.

In Figure 9, we plot the relationship between multipliers and initial policy room levels.

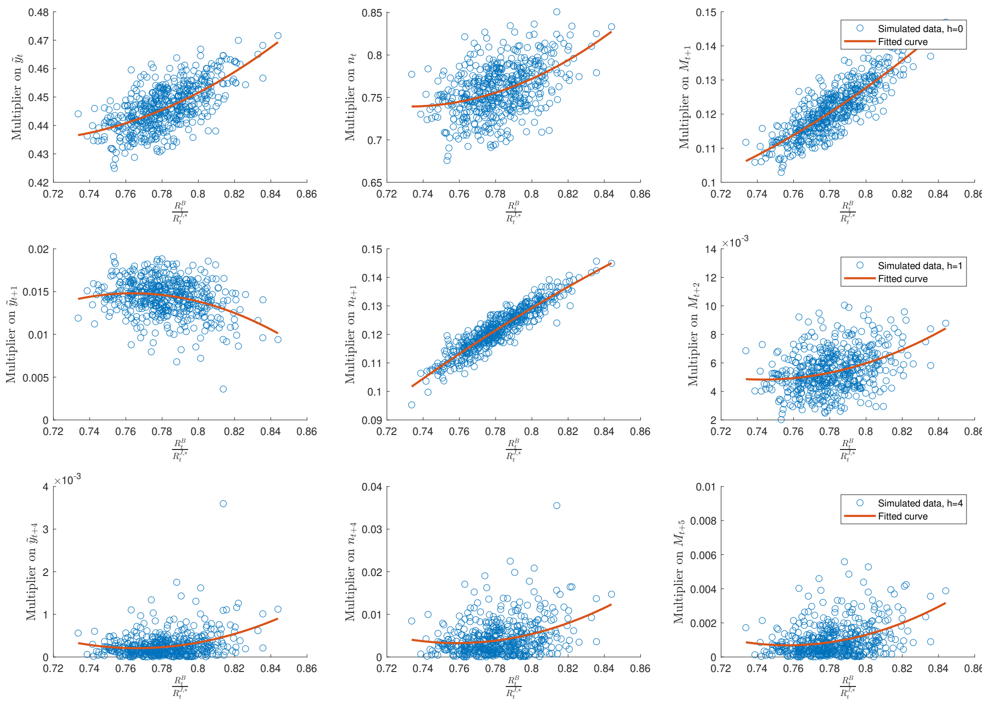


Figure 9: Scatter plot between policy room and monetary policy multipliers.

Notes: Figures plot the relationship between policy room and monetary policy multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t + 1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

The key findings are:

1. At $h = 0$, the multipliers for output and labor positively correlate with initial policy room levels. This effect is due to the higher rate of firm entry (which in turn raises equipment purchases) in response to a monetary shock when initial policy room is larger, consistent with Corollary 1. We will test this channel in the data shortly.

2. At $h = 1$, although the multipliers decline due to the shock's lack of persistence, the positive correlation with the initial policy room remains. This is explained by an increased number of firms in the market and an associated rise in supply.

3. At $h = 4$, multipliers approach zero, attributable to the lack of shock persistence.

In summary, the policy room serves as a sufficient statistic for equilibrium firm entry and is positively correlated with the multipliers for output, labor, and firm entry in response to monetary shocks. Further details can be found in Figures D.13 and D.14[28] in Appendix D, which relate closely to the discussion here.

5 Empirical Analysis

In this section, we empirically test the key model implication from Corollary 1 and Section 4.3 that a higher initial level of the policy room raises the efficacy of monetary policy, since there is a larger room for endogenous responses in firm entry, which further affects both aggregate demand and aggregate supply. In that purpose, in Appendix C.4, we provide two ways to recover the policy room $\frac{R_t^B}{R_t^{J,*}}$, which is unobservable, from the data on the labor markets (e.g., the total number of unemployment) and on the measure of firm participation (e.g., the number of establishements). Here, we focus on the policy room recovered from the measure of firm participation (i.e., Version 2 in Appendix C.2).[29]

Our benchmark local projection à la Jordà (2005)

$$\tilde{y}_{t,t+h} = \sum_{q=1}^{Q} \tilde{y}_{t-q} + \sum_{m=0}^{M} \beta_{0,m}^{(h)} \epsilon_{t-m} + \sum_{n=1}^{N} \beta_{R,n}^{(h)} \widehat{r_{t-m}^{B} - r_{t-m}^{J*}} + \sum_{p=0}^{Q} \beta_{0R,p}^{(h)} \epsilon_{t-p} \times \widehat{r_{t-p-1}^{B} - r_{t-p-1}^{J*}} + u_{t+h|t}$$

is specified by the following components:

[28]Figure D.14 in Appendix D documents the relation between the policy room and the government spending multiplier, which is similar to the case of monetary policy in Figure 9.

[29]We also provide the results based on the measure recovered from labor market variables (i.e., Version 1 in Appendix C.1) in Appendix E.

1. Monetary policy shocks ϵ_t: Acosta (2023)'s extended Romer and Romer (2004) monetary policy shocks. In Appendix E, we present results based on Wieland and Yang (2020)'s extended series of Romer and Romer (2004).

2. Policy room measure $\widehat{r_t^B - r_t^{J*}}$: log-deviation of the policy room $\frac{R_t^B}{R_t^{J,*}}$ from the steady state, recovered by the data on the total number of establishments from the Quarterly Census of Employment and Wages (QCEW) in Appendices C.2 and C.4.

3. Controls and variables of interest: in our benchmark regression, we control current and four lags of federal funds rates. We use log consumption, log output, unemployment rate (%), and log the number of establishments (QCEW) for $\tilde{y}_t$.

4. Number of lags: $Q = M = N = 4$. In Appendix E, we discuss the robustness of our results across different numbers of lags.

Figure 10 shows our benchmark regression result with monetary shocks from Acosta (2023). It displays the impulse response functions of output, consumption, and unemployment to monetary policy shocks and the interaction of monetary policy shocks with the policy room deviation constructed from the firm entry measure (i.e., Version 2 in Appendix C.2).[30]

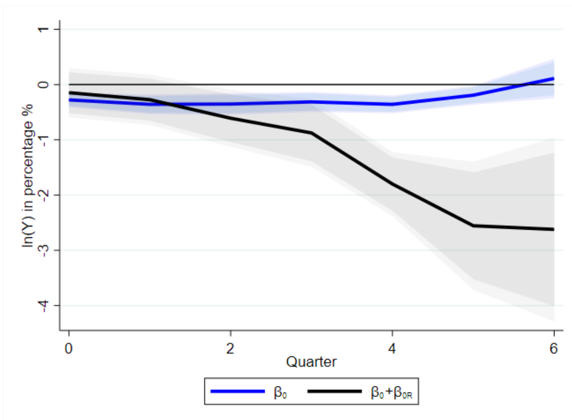
A higher (retrieved) policy room significantly increases the potency of monetary policy shocks, e.g., for output, one standard deviation increase (2 percentage points) in log-policy room,[31] raises a magnitude of output decrease (in %) in response to a one standard deviation tightening shock by around 3 percentage points.

Panel (d) emphasize that this differential effect of monetary policy shocks under different policy room levels works through endogenous firm entry, i.e., the number of establishments drops more with a tightening monetary shock, under a higher policy room measure.

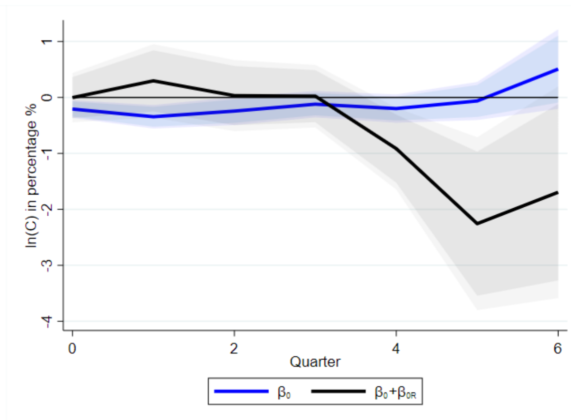
With Additional Controls We add more controls to our benchmark regression and test the robustness of our results. The additional controls are current and four lags of federal funds rates, four lags of oil price growth rate, four lags of long-term interest rate, four lags of consumption growth rate, four lags of GDP deflator, four lags of shadow federal funds rate from Wu and Xia (2016). Figure 11 shows no to little difference from Figure 10 where no additional control is added.

[30]With our benchmark regression relying on the policy room recovered from the number of establishment data from QCEW, we use Acosta (2023) monetary policy shock series since the Wieland and Yang (2020) shock series ends in 2007, thus could not provide us with enough observations for empirical analysis.

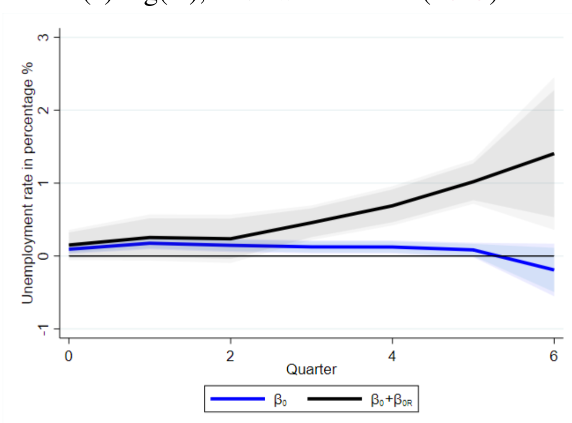
[31]It corresponds 2% increase in the actual policy room measure.



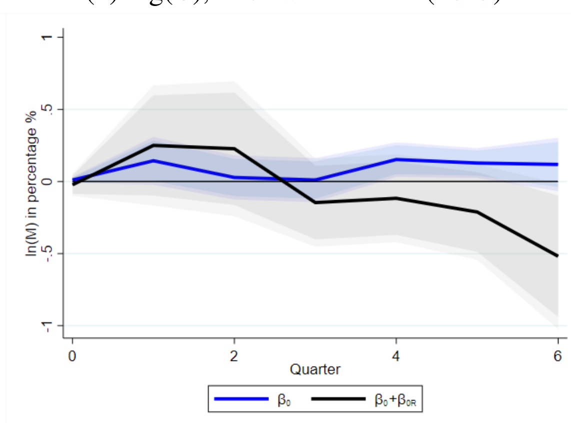
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



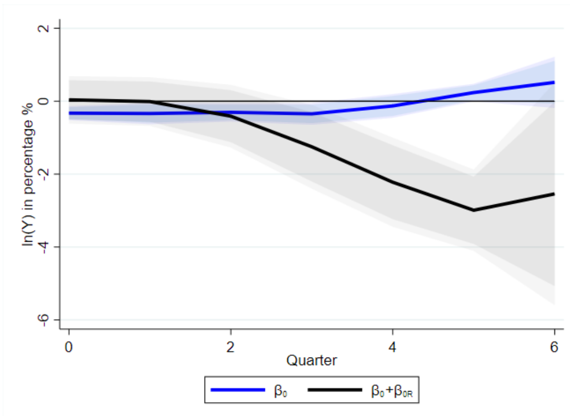
(c) Unemployment rate, in % with Acosta (2023)



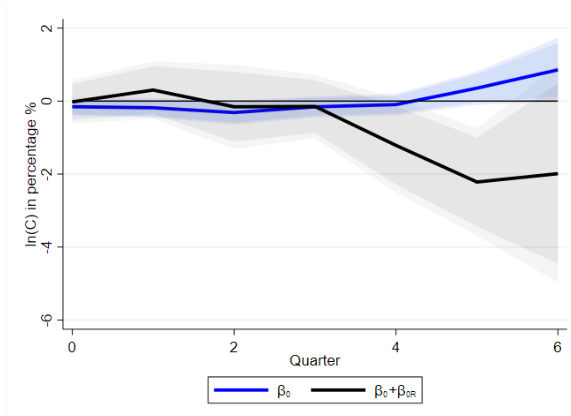
(d) log(M), in % with Acosta (2023)

Figure 10: Local projection: with policy room from Version 2

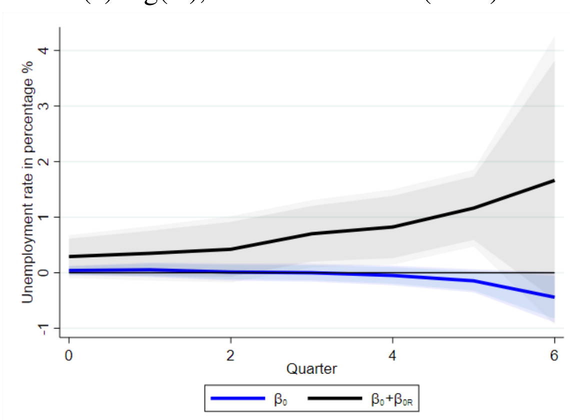
Notes: The impulse response functions are based on the benchmark regression, which controls for current and four lags of federal funds rate. The IRFs are to one positive standard deviation (29 basis points) in monetary policy shock with one standard deviation increase (2 percentage points) in log policy room constructed based on Version 2 (i.e., Appendix C.2). The share of operating firms M, which is used to measure the policy room, is measured by the number of establishments from the Quarterly Census of Employment and Wages (QCEW) dataset.



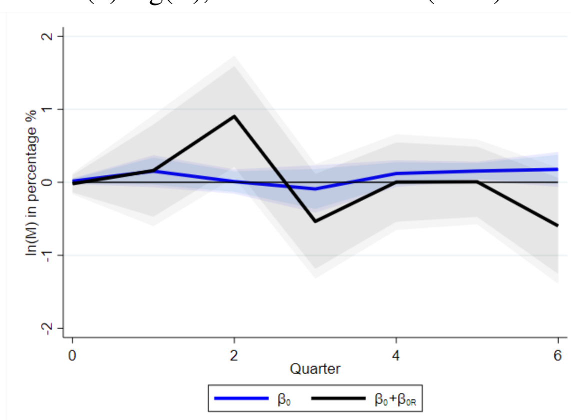
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



(c) Unemployment rate, in % with Acosta (2023)



(d) log(M), in % with Acosta (2023)

Figure 11: Local projection: with policy room from Version 2 and additional controls

Notes: The impulse responses functions are for the local projection with the following additional controls: current and four lags of federal funds rates, four lags of oil price growth rate, four lags of long-term interest rate, four lags of consumption growth rate, four lags of GDP deflator, four lags of shadow federal funds rate from Wu and Xia (2016)

Additional Robustness In Appendix E, we provide the result based on a different method to recover the policy room $\widehat{r_t^B - r_t^{J*}}$, where we use the employment data, e.g., CES National Databases in the Bureau of Labor Statistics (BLS), and monetary policy shocks from Wieland and Yang (2020). The results are still qualitatively similar to Figure 10.

6 Conclusion

This paper develops a macroeconomic framework to understand the contributions of endogenous firm entry to business cycle fluctuations. Based on a dual-industry (i.e., upstream and downstream industries) model, we tractably characterize the dynamics of endogenous firm entry within a New-Keynesian framework. In our model, upstream firms face stochastic fixed entry costs, denominated in the final consumption good. These firms are also constrained by cash-in-advance requirements and depend on capital markets for financing their fixed costs. Downstream firms, on the other hand, are subject to nominal rigidities. Our analysis reveals that demand shocks increase firm profitability and entry, thereby expanding the economy's aggregate supply. In turn, this increased participation stimulates additional demand for the final good, as firms seek to finance their entry via loans. This process initiates a self-reinforcing cycle, rendering the relationship between demand and supply non-separable under general circumstances. As a result, conventionally defined 'supply' and 'demand' shocks can induce comparable patterns of business cycle co-movement. Specifically, supply shifts, resulting from the entry of new firms, lead to disinflationary pressures alongside an increase in output.

Our research identifies a critical threshold for each entry fixed cost level, termed the Satiation Bound (SB). At this threshold, all firms with identical entry fixed costs fully engage in production, rendering monetary policy ineffective in further spurring economic growth through new firm entry. Based on this concept, we introduce a metric known as the "policy room", which represents the difference between the current policy rate and the average SB across firms. It turns out that there is a strong correlation between the rate of firm entry, monetary policy efficacy, and our policy room measure.

We further analyze changes in aggregate variables such as labor, breaking them down into two components: the 'extensive' margin, involving new firm entries, and the 'intensive' margin, related to activities of incumbent firms. We show that a wider policy room makes firm entry decisions more responsive to changes in the policy rate, leading to higher policy multipliers, which we confirm to hold in the data with our empirical exercises. Conversely, when the policy room is narrow, the intensive margin becomes predominant, and the economy's response to shocks is characterized by lower output multipliers and heightened inflation responses. Therefore, we believe that understanding the drivers of firm entry is key to figuring out how demand and supply interact at business cycle frequencies.

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Online Appendix for
Endogenous Firm Entry and the Supply-Side Effects of Monetary Policy

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A Derivation and Proofs

A.1 Detailed Derivation in Section 2.2

Derivation of equations (12) and (13) The price setting of a firm (m, v) is given by

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1-\alpha}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} \left[(P^J_{mv,t})^{-\sigma}\Gamma^J_t\right]^{\frac{1-\alpha}{\alpha}} ,$$

in which we can solve for $P^J_{mv,t}$ as

$$(P^J_{mv,t})^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} (\Gamma^J_t)^{\frac{1-\alpha}{\alpha}} ,$$

from which we obtain

$$P^J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{1}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{(1-\alpha)}{\alpha+\sigma(1-\alpha)}} . \tag{A.1}$$

To get the revenue function $r_{mv,t}$, we start from

$$P^J_{mv,t} J_{mv,t} = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right) W_t \varphi_{mv,t}^{-\frac{1}{\alpha}} J_{mv,t}^{\frac{1}{\alpha}} ,$$

which leads to

$$r_{mv,t} = (1+\zeta^J) P^J_{mv,t} J_{mv,t} = \left(\frac{\sigma}{(\sigma-1)\alpha}\right) W_t N_{mv,t} = (1+\zeta^J) P^J_{mv,t} \left(\frac{P^J_{mv,t}}{P^J_t}\right)^{-\sigma} J_t \tag{A.2}$$

$$= (1+\zeta^J)(P^J_{mv,t})^{1-\sigma}\Gamma^J_t = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{-\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma^J_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} .$$

Finally, we obtain the formula for the profit $\Pi^J_{mv,t}$, which is given by

$$\Pi^J_{mv,t} = r_{mv,t} - W_t N_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} = \frac{\alpha + \sigma(1-\alpha)}{\sigma} r_{mv,t} - R^J_{t-1} P_{t-1} F_{m,t-1} \ .$$

Calculating $P^J_{m,t}$ in (6): the price aggregator for firms of fixed $F_{m,t-1}$ From our notation in (6), we know that among firms with fixed cost $F_{m,t-1}$, a set of operating ones at t would be given by $\Omega_{m,t} = \left\{\varphi_{mv,t} \in \left[\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}, \infty\right]\right\}$. The cumulative distribution function of productivities of upstream firms that decide to produce is $\frac{\Psi(\varphi_{m,t})}{1-\Psi(\varphi^*_{m,t})}$, a truncated Pareto distribution which is itself a Pareto distribution. With the individual firm (m, v)'s pricing equation (A.1), we now can compute the aggregate price of upstream firms with fixed cost $F_{m,t-1}$ as:

$$\left(\frac{P^J_{m,t}}{P_t}\right)^{1-\sigma} = \cancel{M_{m,t}} \cdot \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \frac{\mathrm{d}\,\Psi(\varphi_{mv,t})}{\cancel{1-\Psi(\varphi^*_{m,t})}} \tag{A.3}$$

$$= \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}}^{\infty} \left(\frac{P^J_{mv,t}}{P_t}\right)^{1-\sigma} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}}$$

$$\cdot \left(\frac{\Gamma^J_t}{(P^J_t)^\sigma A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \int_{\max\left\{\varphi^*_{m,t}, \left(\frac{\kappa-1}{\kappa}\right) A_t\right\}}^{\infty} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \mathrm{d}\,\Psi(\varphi_{mv,t})$$

$$= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \max\left\{\frac{\varphi^*_{m,t}}{\left(\frac{\kappa-1}{\kappa}\right) A_t}, 1\right\}^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}}$$

$$= \Theta_1 \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{P^J_t}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}} \tag{A.4}$$

$$\cdot \min\left\{\left(\frac{R^J_{t-1} P_{t-1} F_{m,t-1}}{E_{t-1}\left[\xi_t \cdot \Xi_t\right] \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right\},$$

where we define

$$\Theta_1 = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) .$$

Reexpressing Ξ_t in equation (13) Combining equation (13) with $\Gamma_t^J = (P_t^J)^\sigma Y_t \Delta_t$, we obtain

$$\begin{aligned}\Xi_t &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}}\left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \\ &\quad \cdot \left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t (Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \\ &= \Theta_2 \cdot \left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} P_t (Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}} ,\end{aligned} \tag{A.5}$$

where we define

$$\Theta_2 = \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} .$$

Derivation of P_t^J in (19) We start from the full satiation threshold of the fixed cost F_{t-1}^* defined in Proposition 2:

$$\begin{aligned}F_{t-1}^* &= \frac{E_{t-1}\left[\xi_t \cdot \Xi_t\right]\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J P_{t-1}} \\ &= \Theta_2 E_{t-1}\left[\xi_t\left(\frac{P_t^J}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{W_t}{A_t P_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left[\left(\frac{\kappa-1}{\kappa}\right)A_t\right]^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}}\left(\frac{\Pi_t (Y_t\Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}}{R_{t-1}^J}\right)\right] ,\end{aligned} \tag{A.6}$$

where the second equality is from equation (A.5). From (14) and (A.6), we obtain

$$\varphi_{m,t}^* = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}}\left(\frac{\kappa-1}{\kappa}\right)A_t, \quad R_{m,t-1}^{J,*} = \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-1} R_{t-1}^J . \tag{A.7}$$

From (15), we obtain

$$M_{m,t} = \min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)}, 1\right\} . \tag{A.8}$$

Using equation (A.3) and (A.6), we obtain

$$\begin{aligned}\left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\left(\frac{P_t^J}{P_t}\right)^{\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}}\left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \\ &\quad \cdot \min\left\{\left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)}, 1\right\} .\end{aligned} \tag{A.9}$$

We rearrange equation (6) as:

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \int_0^1 \left(\frac{P_{m,t}^J}{P_t}\right)^{1-\sigma} \mathrm{d}m \\
&= Prob\left(F_{m,t-1} \leq F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} |F_{m,t-1} \leq F_{t-1}^*\right] \\
&\quad + Prob\left(F_{m,t-1} > F_{t-1}^*\right) E_t\left[\left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} |F_{m,t-1} > F_{t-1}^*\right] \qquad \text{(A.10)} \\
&= \cancel{H(F_{t-1}^*)} \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{H(F_{t-1}^*)}} + \cancel{\left[1 - H(F_{t-1}^*)\right]} \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1 - H(F_{t-1}^*)}} \\
&= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1}) + \int_{F_{t-1}^*}^{\infty} \left(\frac{P_{m,t}^J}{P_t^J}\right)^{1-\sigma} \mathrm{d}H(F_{m,t-1}) \,,
\end{aligned}
$$

where $\frac{P_{m,t}^J}{P_t^J}$ is given by (A.9). Plugging (A.9) into (A.10), we obtain

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{1-\sigma} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P_t^J}{P_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left[\int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F_{t-1}^*} 1 \; \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F_{t-1}^*}^{\infty} \left(\frac{F_{m,t-1}}{F_{t-1}^*}\right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right)\right] \,, \qquad \text{(A.11)}
\end{aligned}
$$

which leads to

$$
\begin{aligned}
\left(\frac{P_t^J}{P_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} &= \Theta_1 \cdot \left(\frac{W_t}{P_t A_t}\right)^{\left(\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\left(\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}\right)} \\
&\quad \cdot \left[H\left(F_{t-1}^*\right) + \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + (\omega-1)(\sigma-1)}\right) \cdot \left[1 - H\left(F_{t-1}^*\right)\right]\right] \,. \qquad \text{(A.12)}
\end{aligned}
$$

Rearranging equation (A.12), we finally obtain:

$$
\frac{P_t^J}{P_t} = \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H\left(F_{t-1}^*\right)}\right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)}\right)} \,. \qquad \text{(A.13)}
$$

where we define

$$\Theta_3 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}\right), \quad \Theta_4 = \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right) .$$

Derivation of M_t and L_{t-1} in (20) and (21)

$$\begin{aligned} M_t &= \int_0^1 \int_{v\in\Omega_{m,t}} 1 \, \mathrm{d}v \, \mathrm{d}m = \int_0^1 M_{m,t} \, \mathrm{d}m = \int_0^1 M_{m,t} \cdot dH(F_{m,t-1}) \qquad \text{(A.14)} \\ &= \underbrace{Prob\left(F_{t-1} \le F^*_{t-1}\right)}_{=H(F^*_{t-1})} \cdot 1 + \cancel{Prob\left(F_{t-1} > F^*_{t-1}\right)} \cdot \int_{F^*_{t-1}}^{\infty} \left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}} \frac{\mathrm{d}H(F_{m,t-1})}{\cancel{1-H\left(F^*_{t-1}\right)}} \\ &= 1 - \Theta_M \cdot \left[1 - H\left(F^*_{t-1}\right)\right] , \end{aligned}$$

where

$$\Theta_M = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)} .$$

To derive equation (17), we start from

$$\begin{aligned} \frac{L_{t-1}}{P_{t-1}} &= \frac{\int_0^1 L_{m,t-1} \, \mathrm{d}m}{P_{t-1}} \\ &= Prob\left(F_{m,t-1} \le F^*_{t-1}\right) \cdot \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1} \frac{\mathrm{d}H\left(F_{m,t-1}\right)}{H\left(F^*_{t-1}\right)} \\ &\quad + Prob\left(F_{m,t-1} > F^*_{t-1}\right) \cdot \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \frac{\mathrm{d}H\left(F_{m,t-1}\right)}{1-H\left(F^*_{t-1}\right)} \\ &= \int_{\left(\frac{\omega-1}{\omega}\right)F_{t-1}}^{F^*_{t-1}} F_{m,t-1} \, \mathrm{d}H\left(F_{m,t-1}\right) + \int_{F^*_{t-1}}^{\infty} \left(F^*_{t-1}\right)^{\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1}\right)} \cdot F_{m,t-1}^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}\right)} \mathrm{d}H\left(F_{m,t-1}\right) , \end{aligned}$$

which leads to

$$\begin{aligned} \frac{L_{t-1}}{P_{t-1}} &= F_{t-1} - \left(\frac{\omega}{\omega-1}\right)\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}\right) \cdot F^*_{t-1} \cdot \left[1 - H\left(F^*_{t-1}\right)\right] \\ &= F_{t-1} \cdot \left[1 - \Theta_L \cdot \left[1 - H\left(F^*_{t-1}\right)\right]^{\left(\frac{\omega-1}{\omega}\right)}\right] , \end{aligned}$$

where

$$\Theta_L = \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)} .$$

Derivation of N_t in equation (25) Labor $N_{mv,t}$ employed by a producing upstream firm (m, v) is given by

$$N_{mv,t} = J_{mv,t}^{\frac{1}{\alpha}} \varphi_{mv,t}^{-\frac{1}{\alpha}} = \varphi_{mv,t}^{-\frac{1}{\alpha}} \cdot \left[\left(\frac{P_{mv,t}^J}{P_t^J} \right)^{-\sigma} \cdot J_t \right]^{\frac{1}{\alpha}} \tag{A.15}$$
$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right) A_t} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)}$$
$$\cdot \left(\frac{W_t}{P_t A_t} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{P_t^J}{P_t} \right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(\frac{Y_t \Delta_t}{A_t} \right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}$$
$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\varphi_{mv,t}}{\left(\frac{\kappa-1}{\kappa}\right) A_t} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \tag{A.16}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} ,$$

where we use equation (5) in the second equality, equations (8) and (11) for the third equality, and equation (19) to obtain the fourth one. For convenience we define $H_{t-1} \equiv H\left(F_{t-1}^*\right)$. Now we integrate labor in (A.15) across all producing firms to obtain the aggregate labor N_t. First,

$$N_t = \int_0^1 \int_{v\in\Omega_{m,t}} N_{mv,t} \, \mathrm{d}v \, \mathrm{d}m$$
$$= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa} \right) A_t \right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \tag{A.17}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m$$
$$= \Box \int_0^1 \int_{v\in\Omega_{m,t}} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \, \mathrm{d}v \, \mathrm{d}m ,$$

where

$$\Box = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha} \right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left[\left(\frac{\kappa-1}{\kappa} \right) A_t \right]^{\left(\frac{1-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \tag{A.18}$$
$$\cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{1}{\alpha}} .$$

Now, (35) leads to

$$
\begin{aligned}
N_t &= \square \int_0^1 \int_{\max\left(\varphi^*_{m,t}, \frac{\kappa-1}{\kappa} A_t\right)} \varphi_{mv,t}^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \kappa \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \varphi_{mv,t}^{-(\kappa+1)} \, \mathrm{d}\varphi_{mv,t} \, \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\kappa} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \int_0^1 \max\left(\frac{\varphi^*_{m,t}}{\frac{\kappa-1}{\kappa} A_t}, 1\right)^{\left(-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\alpha+\sigma(1-\alpha)}\right)} \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \int_0^1 \min\left(\left(\frac{F_{m,t-1}}{F^*_{t-1}}\right)^{-\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\sigma-1}}, 1\right) \mathrm{d}m \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\
&= \square \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \\
&= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) [1+\Theta_4 H_{t-1}] \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\
&= \Theta_N \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot (1+\Theta_4 H_{t-1})^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}},
\end{aligned}
\tag{A.19}
$$

where Θ_N is defined in (26).

Equilibrium conditions for downstream firms Plugging equation (28) and the expression for $Q_{t,t+l}$ into (4), we can express the resetting price in (4) in a recursive fashion as

$$
\begin{aligned}
O_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right) \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{Y_t}{A_t}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)} \Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} (1+\Theta_4 H_{t-1})^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \exp\{-u_{c,t}\} \\
&\quad + \beta\theta E_t \left[\exp\{u_{c,t+1} - u_{c,t}\} \cdot \Pi_{t+1}^{\gamma} \cdot O_{t+1}\right],
\end{aligned}
\tag{A.20}
$$

and

$$
V_t = \left(\frac{C_t}{Y_t}\right)^{-1} + \beta\theta \cdot E_t \left[\exp\{u_{c,t+1} - u_{c,t}\} \cdot \Pi_{t+1}^{\gamma-1} \cdot V_{t+1}\right]. \tag{A.21}
$$

We obtain

$$\frac{P_t^*}{P_t} = \frac{O_t}{V_t} . \tag{A.22}$$

Due to price stickiness à la Calvo (1983), the aggregate price level can be recursively expressed as:

$$P_t^{1-\gamma} = (1-\theta)\left(P_t^*\right)^{1-\gamma} + \theta\left(P_{t-1}\right)^{1-\gamma} ,$$

or alternatively as:

$$\frac{P_t^*}{P_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} . \tag{A.23}$$

Plugging equation (A.22) into equation (9) and equation (A.23), we obtain

$$\frac{O_t}{V_t} = \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} , \quad \Delta_t = (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma} + \theta\Pi_t^{\gamma}\Delta_{t-1} .$$

Equilibrium conditions for households We can write F_t^* as a function of H_t by using the cumulative distribution function of fixed costs in (18) and (23):

$$F_t^* = \left[1-H_t\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}A_t\cdot\exp\left\{u_{f,t}\right\} . \tag{A.24}$$

Using the above (A.24), we can rearrange equation (A.6) (i.e., equation about F_t^* as:

$$R_t^J = E_t\left[\xi_{t+1}\cdot\left(\frac{P_{t+1}^J}{P_{t+1}}\right)^{\left(\frac{\sigma}{\alpha+\sigma(1-\alpha)}\right)}\left(\frac{w_{t+1}}{P_{t+1}A_{t+1}}\right)^{\left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)}\frac{1}{\tilde{Y}}\Pi_{t+1}GA_{t+1}\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{1}{\alpha+\sigma(1-\alpha)}\right)}\right]$$
$$\cdot\left(\frac{\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left[1-H_t\right]^{\frac{1}{\omega}}\cdot\exp\left\{-u_{f,t}\right\} . \tag{A.25}$$

Plugging (27) and (28) into the above (A.25), we obtain:

$$R_t^J = \left(\frac{\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left(1+\Theta_4H_t\right)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}\cdot\left(1-H_t\right)^{\frac{1}{\omega}} \tag{A.26}$$
$$\cdot E_t\left[\xi_{t+1}\Pi_{t+1}\left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{Y_{t+1}}{A_{t+1}}}\right)\left(\frac{\frac{Y_{t+1}}{A_{t+1}}}{\tilde{Y}}\right)\left(\frac{Y_{t+1}\Delta_{t+1}}{A_{t+1}}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\cdot GA_{t+1}\cdot\exp\left\{-(u_{f,t}+u_{c,t+1})\right\}\right] .$$

Finally, we can rearrange the Euler equation in (1), using (30) as follows:

$$\frac{1}{R_t^J} = \beta E_t \left[\frac{\left(\frac{C_t}{Y_t}\right)}{\left(\frac{C_{t+1}}{Y_{t+1}}\right) \widetilde{GY}_{t+1} GA_{t+1} \Pi_{t+1}} \cdot \exp\left\{u_{c,t+1} - u_{c,t}\right\} \right] , \tag{A.27}$$

where $\widetilde{GY}_{t+1} = \frac{Y_{t+1}}{Y_{t+1}} \frac{A_t}{A_{t+1}}$ and $GA_{t+1} = \frac{A_{t+1}}{A_t}$. Combining equation (A.26) and equation (A.27), we obtain

$$\exp\left\{u_{f,t} + u_{c,t}\right\} = \beta \left(\frac{\Theta_2 \cdot \Theta_N^{\frac{1}{\eta}} \cdot \Theta_3^{\overline{\frac{\sigma}{(\sigma-1)\alpha}}}}{\left(\frac{\omega-1}{\omega}\right) \phi_f} \right) \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left(1 + \Theta_4 H_t\right)^{\left(\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{\eta(1-\sigma)\alpha}\right)}$$
$$\cdot \left(1 - H_t\right)^{\frac{1}{\omega}} \cdot \left(\frac{\frac{C_t}{A_t}}{\tilde{Y}} \right) \cdot E_t \left[\left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\left(\frac{\eta+1}{\eta\alpha}\right)} \right] . \tag{A.28}$$

Flexible price equilibrium Plugging (34) into (19), we obtain

$$\frac{W_t}{P_t A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1} \right)^{-1} \cdot \left(\frac{Y_t \Delta_t}{A_t} \right)^{\frac{\alpha-1}{\alpha}} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_{t-1}} \right]^{\left(\frac{\alpha+\sigma(1-\alpha)}{1-\sigma}\right)} . \tag{A.29}$$

Plugging (19) and (A.29) into (A.6) (i.e., equation about the cutoff fixed cost F_t^*), and based on the fact that there is no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain:

$$F_t^* = \Theta_2 \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1} \right)^{-1} \cdot \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_t} \right] E_t \left[\xi_{t+1} \left(\frac{\Pi_{t+1} Y_{t+1}}{R_t^J} \right) \right] . \tag{A.30}$$

By the definition of the distribution function of the fixed costs (see eq. (18)), we express:

$$\left[1 - H_t\right]^{-\frac{1}{\omega}} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) F_t} = \frac{F_t^*}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f \cdot \tilde{Y} A_t \cdot \exp\left\{u_{f,t}\right\}} . \tag{A.31}$$

Plugging equation (A.31) into equation (A.30), we obtain:

$$1 = \left(\frac{\beta \Theta_2}{\left(\frac{\omega-1}{\omega}\right) \cdot \phi_f} \right) \cdot \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1} \right)^{-1} \cdot \left(\frac{\kappa-1}{\kappa} \right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)} \cdot \left[\frac{\Theta_3}{1 + \Theta_4 \cdot H_t} \right]$$
$$\cdot \left[1 - H_t\right]^{\frac{1}{\omega}} \cdot E_t \left[\left(\frac{\tilde{Y}_t}{\tilde{Y}} \right) \left(\frac{\frac{C_t}{Y_t}}{\frac{C_{t+1}}{Y_{t+1}}} \right) \cdot \exp\left\{u_{c,t+1} - (u_{f,t} + u_{c,t})\right\} \right] . \tag{A.32}$$

Finally, plugging (34) into (28) and based on no price dispersion under flexible prices, i.e., $\Delta_t = 1$, we obtain

$$\frac{Y_t}{A_t} = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma - 1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)} \Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)} \Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}} \cdot \left(\frac{C_t}{A_t}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$
$$\cdot (1+\Theta_4 H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}} \cdot \exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right) \cdot u_{c,t}\right\} . \tag{A.33}$$

From (A.32) and (A.33), we can see that the flexible price equilibrium is money-neutral.

A.2 Calibration of (κ, ω) in Section 3.1

Following intutions of Bernard et al. (2003), we calculate the model-implied standard deviation of revenues and productivities of operating upstream firms.

Derivations on the cross-sectional standard deviations of sales and productivities We start from the formula for the revenue $r_{mv,t}$ generated by a firm (m, v) in (A.2):

$$r_{mv,t} = (1+\zeta^J)\left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} W_t^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} (\Gamma_t^J)^{\frac{1}{\alpha+\sigma(1-\alpha)}} \varphi_{mv,t}^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} , \tag{A.34}$$

where

$$\varphi^*_{m,t} = \left(\frac{R^J_{t-1} P_{t-1} F_{m,t-1}}{E_{t-1}[\xi_t \cdot \Xi_t]}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}} . \tag{A.35}$$

We can calculate the cross-sectional standard deviation of an individual firm's revenue and productivity by calculating the variance:

$$\begin{aligned}
\sigma^2(\log r_{mv,t}) &= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \sigma^2(\log \varphi_{mv,t}) \\
&= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}} + \log\varphi^*_{m,t}\right) \\
&= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}}\right) + \sigma^2(\log\varphi^*_{m,t})\right] ,
\end{aligned} \tag{A.36}$$

where for the second line we use the property that (i) $\phi_{mv,t}|\phi_{mv,t} \geq \phi^*_{m,t}$ follows a Pareto distribution; (ii) distributions of productivities and fixed costs are independent of each other.

Therefore,

$$\sigma^2(\log r_{mv,t}) = \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \left[\sigma^2\left(\log\frac{\varphi_{mv,t}}{\varphi^*_{m,t}}\right) + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2 \sigma^2(\log F_{m,t-1})\right]$$
$$= \left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)^2 \left[\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2 \frac{1}{\omega^2}\right],$$

which implies

$$\sigma(\log r_{mv,t}) = \frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2 \frac{1}{\omega^2}},$$

and

$$\sigma(\log \varphi_{mv,t}) = \sqrt{\frac{1}{\kappa^2} + \left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1}\right)^2 \frac{1}{\omega^2}}.$$

Revenue heterogeneity in our model With $\kappa = \omega = 3.4$, our model predicts the standard deviation of upstream firms' revenues to be 0.44. The residual variability in Bernard et al. (2003) stem from some factors we do not account for, such as taste heterogeneity or different demand weights for product types. Additionally, their estimates are based on U.S. manufacturing plants, whereas our framework focuses on upstream firms.

Regarding productivity variability, the standard deviation of log productivity for operating upstream firms in our model becomes 0.4 when $\kappa = \omega = 3.4$. According to Bernard et al. (2003), their model-generated standard deviation of log value-added per worker is 0.35, while the empirical figure stands at 0.75.[1] Given the potential for measurement errors, our calibration is closely aligned with their model-generated moment and falls within a plausible range.

[1] Bernard et al. (2003) note that some degree of under-prediction could result from measurement errors in Census data.

A.3 Detailed Derivation in Section 4.2

Intensive vs. extensive margin labor adjustments: derivation of (37) From (35), (A.18), and (A.19), we know that the aggregate labor N_t can be written as

$$N_t = \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \tag{A.37}$$

$$\begin{aligned} &\cdot \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right] \\ &= \Theta_{DN} \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \underbrace{\left[H_{t-1} + \frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}(1-H_{t-1})\right]}_{\equiv SN_t^I} \\ &= \Theta_{DN} \left[\frac{\Theta_3}{1+\Theta_4 \cdot H_{t-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_t\Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \cdot SN_t^I \,, \end{aligned} \tag{A.38}$$

where

$$\Theta_{DN} \equiv \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right). \tag{A.39}$$

From (A.37), we obtain for $\forall\iota$

$$\frac{N_{t+\iota}-N_t}{N_t} = \underbrace{\left[\frac{1+\Theta_4 \cdot H_{t-1}}{1+\Theta_4 \cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}} - 1}_{=g^{\text{Density}}_{t,t+\iota}} \tag{A.40}$$

$$+ \left\{ 1 + \underbrace{\left[\left[\frac{1+\Theta_4 \cdot H_{t-1}}{1+\Theta_4 \cdot H_{t+\iota-1}}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \left(\frac{Y_{t+\iota}\Delta_{t+\iota}/A_{t+\iota}}{Y_t\Delta_t/A_t}\right)^{\frac{1}{\alpha}} - 1\right]}_{=g^{\text{Density}}_{t,t+\iota}} \right\} \cdot \underbrace{\frac{SN^E_{t,t+\iota}}{SN^I_t}}_{\equiv g^{\text{Entry}}_{t,t+\iota}} . \tag{A.41}$$

Therefore, by (36) and the definition of the decomposition in (37), we obtain

$$g_{t,t+\iota}^{\text{Entry}} \equiv \frac{SN_{t,t+\iota}^{E}}{SN_{t}^{I}} = \frac{(H_{t+\iota-1} - H_{t-1}) + \dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + (\omega-1)(\sigma-1)}(H_{t-1} - H_{t+\iota-1})}{H_{t-1} + \dfrac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)] + (\omega-1)(\sigma-1)}(1 - H_{t-1})}\ , \tag{A.42}$$

which proves equation (38).

B Summary of Equilibrium Conditions

B.1 Sticky Price Equilibrium (i.e., Original Model)

$$
\begin{aligned}
\exp\left\{u_{f,t}+u_{c,t}\right\} &= \beta\left(\frac{\Theta_2\cdot\Theta_N^{\frac{1}{\eta}}\cdot\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left(1+\Theta_4 H_t\right)^{\left(\frac{\alpha+\sigma(1-\alpha)+\sigma\eta}{\eta(1-\sigma)\alpha}\right)} \\
&\quad \cdot\left(1-H_t\right)^{\frac{1}{\omega}}\cdot\left(\frac{\tilde{C}_t}{\tilde{Y}}\right)\cdot E_t\left[\left(\tilde{Y}_{t+1}\Delta_{t+1}\right)^{\left(\frac{\eta+1}{\eta\alpha}\right)}\right] \\
\frac{1}{R_t^J} &= \beta E_t\left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}GA_{t+1}\Pi_{t+1}}\cdot\exp\left\{u_{c,t+1}-u_{c,t}\right\}\right] \\
\frac{\tilde{C}_t}{\tilde{Y}_t} &= 1-\phi_g\cdot\exp\left\{u_{g,t}\right\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot\left[1-H_t\right]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\left\{u_{f,t}\right\} \\
O_t &= \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}_t^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta_t^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}\left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}\exp\left\{-u_{c,t}\right\} \\
&\quad +\beta\theta E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma}\cdot O_{t+1}\right] \\
V_t &= \left(\frac{\tilde{C}_t}{\tilde{Y}_t}\right)^{-1}+\beta\theta\cdot E_t\left[\exp\left\{u_{c,t+1}-u_{c,t}\right\}\cdot\Pi_{t+1}^{\gamma-1}\cdot V_{t+1}\right] \\
\frac{O_t}{V_t} &= \left(\frac{1-\theta}{1-\theta\cdot\Pi_t^{\gamma-1}}\right)^{\frac{1}{\gamma-1}} \\
\Delta_t &= (1-\theta)\left(\frac{O_t}{V_t}\right)^{-\gamma}+\theta\Pi_t^{\gamma}\Delta_{t-1} \\
R_t^J &= R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\tau_y}\cdot\exp\left\{\varepsilon_{r,t}\right\},\ \varepsilon_{r,t}\sim N\left(0,\sigma_r^2\right) \\
\tilde{F}_t^* \equiv \frac{F_t^*}{A_t} &= \left[1-H_t\right]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}\cdot\exp\left\{u_{f,t}\right\} \\
R_t^{J,*} &= \left(\frac{\omega}{\omega+1}\right)\cdot\left(1-H_t\right)^{-\frac{1}{\omega}}\cdot R_t^B \\
N_t &= \Theta_N\cdot\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\alpha}}\cdot\left(1+\Theta_4 H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} \\
g_{t,t+1}^{Density} &= \left[\frac{1+\Theta_4\cdot H_{t-1}}{1+\Theta_4\cdot H_t}\right]^{\left(\frac{\sigma}{(\sigma-1)\alpha}\right)}\left(\frac{\tilde{Y}_{t+1}\Delta_{t+1}}{\tilde{Y}_t\Delta_t}\right)^{\frac{1}{\alpha}}-1 \\
g_{t,t+1}^{Entry} &= \frac{\left(H_t-H_{t-1}\right)+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\left(H_{t-1}-H_t\right)}{H_{t-1}+\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\left(1-H_{t-1}\right)} \\
\frac{W_t}{P_t A_t} &= \Theta_N^{\frac{1}{\eta}}\left(\tilde{C}_t\right)\left(\tilde{Y}_t\Delta_t\right)^{\frac{1}{\eta\alpha}}\left(1+\Theta_4 H_{t-1}\right)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}}\cdot\exp\left\{-u_{c,t}\right\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{P_t^J}{P_t} &= \Theta_N^{\frac{1}{\eta}} \Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\tilde{C}_t\right) \left(\tilde{Y}_t \Delta_t\right)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)} \left(1+\Theta_4 H_{t-1}\right)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}} \cdot \exp\left\{-u_{c,t}\right\} \\
M_{t+1} &= 1-\Theta_M \cdot [1-H_t] \\
\frac{L_t/P_t}{\bar{Y}_t} &= \phi_f \cdot \left[1-\Theta_L \cdot [1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right] \cdot \exp\left\{u_{f,t}\right\} \\
GA_t &= (1+\mu) \cdot \exp\left\{u_{a,t}\right\}
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
u_{a,t} &= \rho_a \cdot u_{a,t-1} + \varepsilon_{a,t},\ \varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right) \\
u_{c,t} &= \rho_c \cdot u_{c,t-1} + \varepsilon_{c,t},\ \varepsilon_{c,t} \sim N\left(0, \sigma_c^2\right) \\
u_{g,t} &= \rho_g \cdot u_{g,t-1} + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right) \\
u_{f,t} &= \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t},\ \varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)
\end{aligned}
$$

Parameters:

$$
\begin{aligned}
\Theta_1 &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
\Theta_2 &= \frac{\alpha+\sigma(1-\alpha)}{\alpha(\sigma-1)} \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{-\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\sigma(1-\alpha)}} \\
\Theta_3 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}{\Theta_1\omega(\sigma-1)}\right) \\
\Theta_4 &= \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{\omega(\sigma-1)}\right) \\
\Theta_N &= \left(\frac{(1+\zeta^J)^{-1}\sigma}{(\sigma-1)\alpha}\right)^{\left(\frac{-\sigma}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{\sigma-1}{\alpha+\sigma(1-\alpha)}\right)} \left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}\right) \\
&\quad \cdot \left(\frac{\omega(\sigma-1)}{\kappa[\alpha+\sigma(1-\alpha)]+(\omega-1)(\sigma-1)}\right) \Theta_3^{\left(\frac{\sigma}{\alpha(\sigma-1))}\right)} > 0 \\
\Theta_M &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+\omega(\sigma-1)} \\
\Theta_L &= \frac{\kappa[\alpha+\sigma(1-\alpha)]}{\kappa[\alpha+\sigma(1-\alpha)]+(\sigma-1)(\omega-1)}
\end{aligned}
$$

B.2 Flexible Price Equilibrium

$$1 = \left(\frac{\beta\Theta_2}{\left(\frac{\omega-1}{\omega}\right)\cdot\phi_f}\right)\cdot\left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-1}\cdot\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\cdot\left[\frac{\Theta_3}{1+\Theta_4\cdot H_t}\right][1-H_t]^{\frac{1}{\omega}}$$

$$\cdot E_t\left[\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)\left(\frac{\tilde{C}_t/\tilde{Y}_t}{\tilde{C}_{t+1}/\tilde{Y}_{t+1}}\right)\cdot\exp\left\{u_{c,t+1}-(u_{f,t}+u_{c,t})\right\}\right]$$

$$\tilde{Y}_t = \left(\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right)^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}\Theta_N^{-\left(\frac{\alpha}{(1-\alpha)\eta+1}\right)}\Theta_3^{-\frac{\eta[\alpha+\sigma(1-\alpha)]}{[(1-\alpha)\eta+1](\sigma-1)}}\cdot\tilde{C}_t^{-\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)}$$

$$\cdot(1+\Theta_4H_{t-1})^{-\frac{(1+\eta)[\alpha+\sigma(1-\alpha)])}{(1-\sigma)[(1-\alpha)\eta+1]}}\cdot\exp\left\{\left(\frac{\eta\alpha}{(1-\alpha)\eta+1}\right)\cdot u_{c,t}\right\}$$

$$\frac{\tilde{C}_t}{\tilde{Y}_t} = 1-\phi_g\cdot\exp\{u_{g,t}\}-\phi_f\cdot\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{-1}\cdot\left[1-\Theta_L\cdot[1-H_t]^{\left(\frac{\omega-1}{\omega}\right)}\right]\cdot\exp\{u_{f,t}\}$$

$$\tilde{F}_t^* \equiv \frac{F_t^*}{A_t} = [1-H_t]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}\cdot\exp\{u_{f,t}\}$$

$$R_t^J = R^J\cdot\left(\frac{\Pi_t}{\Pi}\right)^{\tau_\pi}\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)^{\tau_y}\cdot\exp\{\varepsilon_{r,t}\}$$

$$R_t^{J,*} = \left(\frac{\omega}{\omega+1}\right)\cdot(1-H_t)^{-\frac{1}{\omega}}\cdot R_t^B$$

Shock processes:

$$GA_t = (1+\mu)\cdot exp\{u_{a,t}\}$$

$$u_{a,t} = \rho_a\cdot u_{a,t-1}+\varepsilon_{a,t}$$

$$u_{c,t} = \rho_c\cdot u_{c,t-1}+\varepsilon_{c,t}$$

$$u_{g,t} = \rho_g\cdot u_{g,t-1}+\varepsilon_{g,t}$$

$$u_{f,t} = \rho_f\cdot u_{f,t-1}+\varepsilon_{f,t}$$

$$\varepsilon_{c,t} \sim N\left(0,\sigma_c^2\right)$$

$$\varepsilon_{a,t} \sim N\left(0,\ \sigma_a^2\right)$$

$$\varepsilon_{g,t} \sim N\left(0,\ \sigma_g^2\right)$$

$$\varepsilon_{f,t} \sim N\left(0,\ \sigma_f^2\right)$$

$$\varepsilon_{r,t} \sim N\left(0,\sigma_r^2\right)$$

B.3 Steady State Conditions

$$R^B = \beta^{-1}(1+\mu)\Pi$$

$$\Delta = \left(\frac{1-\theta}{1-\theta\Pi^{\gamma}}\right)\left(\frac{1-\theta\Pi^{\gamma-1}}{1-\theta}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$\frac{\Theta_3\cdot[1-H]^{\frac{1}{\omega}}}{1+\Theta_4\cdot H} = \left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{(1-\sigma)(1-\alpha)}{\alpha+\sigma(1-\alpha)}\right)}\left[\frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\right]\left[\frac{1-\theta\Pi^{\gamma}}{1-\theta\Pi^{\gamma-1}}\right]\left[\frac{1-\beta\theta\Pi^{\gamma-1}}{1-\beta\theta\Pi^{\gamma}}\right]\left(\frac{\left(\frac{\omega-1}{\omega}\right)\phi_f}{\beta\cdot\Theta_2}\right)$$

$$\tilde{Y} = \frac{\left(\frac{\beta\Theta_2\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\sigma}{(\sigma-1)\alpha}}}{\left(\frac{\omega-1}{\omega}\right)\phi_f}\right)^{-\left(\frac{\eta\alpha}{\eta+1}\right)}\left(\frac{\kappa-1}{\kappa}\right)^{\left(\frac{-\eta\alpha(\sigma-1)(1-\alpha)}{[\alpha+\sigma(1-\alpha)](\eta+1)}\right)}(1+\Theta_4 H)^{-\frac{[\alpha+\sigma(1-\alpha)]+\sigma\eta}{(\eta+1)(1-\sigma)}}(1-H)^{-\frac{\eta\alpha}{\omega(\eta+1)}}}{\Delta\cdot\left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]^{\left(\frac{\eta\alpha}{\eta+1}\right)}}$$

$$\tilde{C} = \left[1-\phi_g-\phi_f\cdot\left[1-\Theta_L\cdot[1-H]^{\left(\frac{\omega-1}{\omega}\right)}\right]\right]\cdot\tilde{Y}$$

$$M = 1-\Theta_M\cdot[1-H]$$

$$\tilde{F}^* = [1-H]^{-\frac{1}{\omega}}\left(\frac{\omega-1}{\omega}\right)\phi_f\cdot\tilde{Y}$$

$$R^{J,*} = \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}}\cdot\beta^{-1}(1+\mu)\Pi$$

$$\frac{R^{J,*}}{R^B} = \left(\frac{\omega}{\omega+1}\right)\cdot(1-H)^{-\frac{1}{\omega}}$$

$$N = \Theta_N\cdot\tilde{Y}^{\frac{1}{\alpha}}\cdot\Delta^{\frac{1}{\alpha}}\cdot(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}}$$

$$\frac{W}{PA} = \Theta_N^{\frac{1}{\eta}}\tilde{C}\tilde{Y}^{\frac{1}{\eta\alpha}}\Delta^{\frac{1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{\alpha+\sigma(1-\alpha)}{\eta(1-\sigma)\alpha}}$$

$$\frac{P_t^J}{P_t} = \Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{C}(\tilde{Y}\Delta)^{\left(\frac{(1-\alpha)\eta+1}{\eta\alpha}\right)}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}$$

$$O = \frac{(1+\zeta^T)^{-1}\gamma}{\gamma-1}\cdot\frac{\Theta_N^{\frac{1}{\eta}}\Theta_3^{\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}}\tilde{Y}^{\left(\frac{\eta+1}{\eta\alpha}\right)}\Delta^{\frac{(1-\alpha)\eta+1}{\eta\alpha}}(1+\Theta_4 H)^{\frac{(1+\eta)[\alpha+\sigma(1-\alpha)]}{\eta(1-\sigma)\alpha}}}{1-\beta\theta\Pi^{\gamma}}$$

$$V = \frac{\left(\frac{\tilde{C}}{\tilde{Y}}\right)^{-1}}{1-\beta\theta\Pi^{\gamma-1}}$$

$$\frac{L/P}{\bar{Y}} = \phi_f\left[1-\Theta_L(1-H)^{\frac{\omega-1}{\omega}}\right]$$

C Estimation of Satiation Measure H_t and the Policy Room

Since the satiation measure H_t and the policy room $\frac{R_t^B}{R_t^{J,*}}$ are not observable, we provide two ways to back out those two unobservable measures from the data. If we recover H_t series from observable data, we can easily recover the policy room measure as well, as these two are tightly related.

C.1 Version 1: Estimation of Satiation Measure H_t

First, we can divide equation (25) by its steady-state expression to obtain:

$$\frac{N_t}{N} = \left(\frac{\tilde{Y}_t}{\tilde{Y}} \cdot \frac{\Delta_t}{\Delta}\right)^{\frac{1}{\alpha}} \cdot \left(\frac{1+\Theta_4 H_{t-1}}{1+\Theta_4 H}\right)^{\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}} .$$

Taking logs, we obtain:

$$\hat{n}_t = \frac{1}{\alpha}\left(\hat{\tilde{y}}_t + \widehat{\log(\Delta_t)}\right) + \left(\frac{\alpha+\sigma(1-\alpha)}{(1-\sigma)\alpha}\right) \widehat{\log(1+\Theta_4 H_{t-1})}. \quad \text{(C.1)}$$

We now proceed by replacing $\hat{n}_t$, $\hat{\tilde{y}}_t$ and $\widehat{\log(\Delta_t)}$ with the HP-filtered empirical estimates based on data on employment, real GDP, and price dispersion, respectively.[2] Once we have these empirical estimates, we plug them into the following equation (C.2), through which we obtain an estimate of $\widehat{\log(1+\Theta_4 H_{t-1})}$ as:

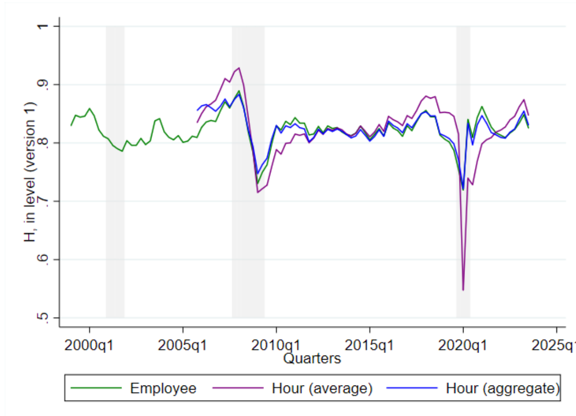
$$\text{Estimate}\left(\widehat{\log(1+\Theta_4 H_{t-1})}\right) = \left(\frac{(1-\sigma)\alpha}{\alpha+\sigma(1-\alpha)}\right)\left[\hat{n}_t - \frac{1}{\alpha}\left(\hat{\tilde{y}}_t + \widehat{\log(\Delta_t)}\right)\right], \quad \text{(C.2)}$$

which leads to

$$\text{Estimate}(H_{t-1}) = \frac{1}{\Theta_4} \cdot \left[\exp\left\{\text{Estimate}\left(\widehat{\log(1+\Theta_4 H_{t-1})}\right) + \log(1+\Theta_4 H)\right\} - 1\right].$$

For the data on N_t, we use (i) the number of employees, (ii) average weekly hour, (iii) index of average weekly hour, all from CES National Databases in the Bureau of Labor Statistics (BLS). Figure C.1 depicts H_t series recovered from this method. We can observe that H_t is hugely procyclical.

[2] Notice that we HP-filter the logs of each variable, not their levels.



(a) HP filter

Figure C.1: The satiation measure H_t: Version 1

Notes: The three series are estimated based on three sources of information on the employment N. Employee: number of employees. Hour (average): average weekly hour. Hour (aggregate): aggregate weekly hour, in thousands. All from BLS CES National Databases.

C.2 Version 2: Estimation of Satiation Measure H_t

Instead of relying on the employment data, here we directly use the data on the number of establishments from the Quarterly Census of Employment and Wages (QCEW), which we use as a proxy for firm entry M_{t+1}.[3] Dividing equation (15) by its steady-state version, we obtain:

$$\frac{M_{t+1}}{M} = \frac{1 - \Theta_M \cdot [1 - H_t]}{1 - \Theta_M \cdot [1 - H]}$$

Taking logs on both sides, with $m_t \equiv \log M_t$:

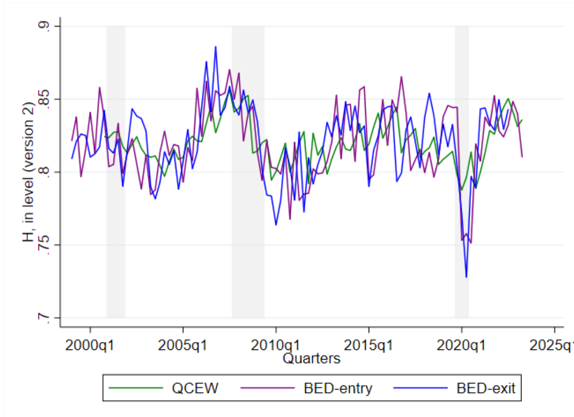
$$\hat{m}_{t+1} = \log\left(1 - \widehat{\Theta_M \cdot [}1 - H_t]\right) \tag{C.3}$$

so we can back out H_t as:

$$H_t = -\frac{1}{\Theta_M} \cdot [1 - \Theta_M - \exp\left\{\hat{m}_{t+1} + \log\left(1 - \Theta_M \cdot [1 - H]\right)\right\}]$$

[3] As seen in Figure C.2, we additionally back out H_t series based on different measures of firm entry: the number of establishments based on firm entry (exit), found in the Business Employment Dynamics (BED) of BLS.

where $\hat{m}_{t+1}$ is the HP-filtered data of the log of firm participation. The spikiness of estimated H under Version 2 comes from the spikiness of the number of establishments data.



(a) HP filter

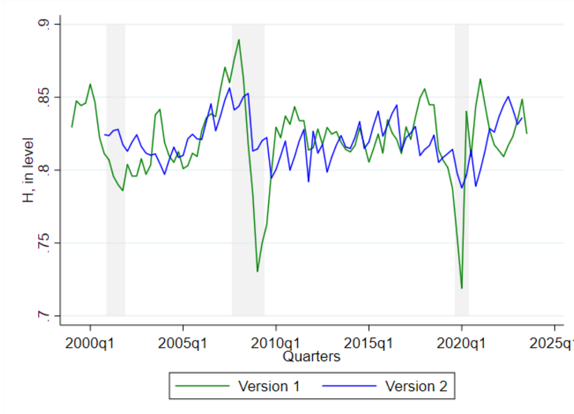
Figure C.2: The satiation measure H_t: Version 2

Notes: The time series are estimated based on three sources of information on the operating firms M. The green line is based on the number of establishments in the QCEW database. The purple (blue) line is the number of establishments based on firm entry (exit) information in BED. The equation is

$$\text{Number of establishment} = \frac{\text{number of establishments with employment gains (loss)}}{\text{percentage of establishments with employment gains (loss)}}.$$

Comparison Figure C.3 compares H_t series recovered by Version 1 (based on the number of employees) and Version 2 (the number of establishments from the QCEW database). Version 1 generates a more volatile H_t series: as our model lacks physical capital, using the formula for labor demand (i.e., equation (25)) to recover the satiation measure H_t might overemphasize the role of H_t in driving labor demand fluctuations and lead to more volatile H_t time series.

In our baseline empirical specification in Section 5, we will use Version 2, based on the number of establishments from the QCEW database, as a benchmark. We provide results based on Version 1 (with the total number of employees) measure of H_t series as necessary robustness checks in Appendix E.



(a) HP filter

Figure C.3: Satiation Measure H_t: Version 1 vs. Version 2

Notes: The H time series within each panel are estimated using two different methods. The green line is based on Version 1 with N measured by the total number of employees. The blue line is based on Version 2 with M measured by the number of establishments from the QCEW database.

C.3 Estimation of the Fixed Cost Process: ϕ_f, ρ_f, and σ_f

We start from:

$$\left(\frac{L_t}{P_t Y_t}\right)\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right) = \phi_f \cdot \left[1 - \Theta_L \cdot (1 - H_t)^{\frac{\omega - 1}{\omega}}\right] \cdot \exp\left(u_t^f\right) \tag{C.4}$$

where the left-hand side is written in the current (private) loan-to-GDP ratio and the output gap. Taking logs and rearranging, we obtain:

$$u_t^f = \log\left(\frac{L_t}{P_t Y_t}\right) + \tilde{y}_t - \log(\phi_f) - \log\left[1 - \Theta_L \cdot (1 - H_t)^{\frac{\omega - 1}{\omega}}\right] \tag{C.5}$$

Once we have an estimate for H_t series and ϕ_f, we can back out u_t^f from equation (C.5) and estimate the AR(1) process parameters ρ_f and σ_f as follows:

$$u_{f,t} = \rho_f \cdot u_{f,t-1} + \varepsilon_{f,t}, \quad \varepsilon_{f,t} \sim N\left(0, \sigma_f^2\right) \tag{C.6}$$

Finally, note that we could estimate ϕ_f using equation (C.5), equation (C.6) and $E\left[\varepsilon_{f,t}\right]$, because H_t is a function of H, which is a (nonlinear) function of ϕ_f.

Figure C.4 plots the time series of nominal private debt to nominal GDP ratio measured by debt securities and loans to GDP ratio for nonfinancial corporate business, which we use as a proxy for $\frac{L_t}{P_t Y_t}$.[4]

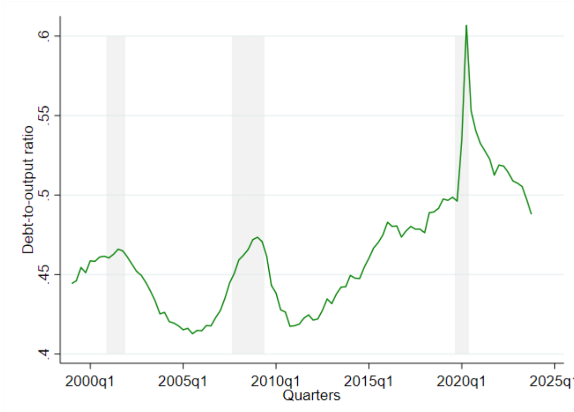


Figure C.4: Private Loan-to-output ratio

C.3.1 Detailed Estimation Procedure

Step 1 We construct an initial guess for ϕ_f around $H_t = 1$ from equation (C.4), as:

$$\hat{\phi}_f = \text{Average}\left\{\left(\frac{L_t}{P_t Y_t}\right)\left(\frac{\tilde{Y}_t}{\tilde{Y}}\right)\right\}$$

Notice that if $\tilde{y}_t$ is the HP-filtered series of the log-output, $\frac{\tilde{Y}_t}{\tilde{Y}}$ is its exponential value.

Step 2 Compute H_t (either from Version 1 or Version 2) using the above guessed $\hat{\phi}_f$.

Step 3 By combining equations (C.5) and (C.6), we obtain:

$$\tilde{u}_t^f = (1-\rho_f)\cdot\log(\phi_f) + \rho_f \cdot \tilde{u}_{t-1}^f + \varepsilon_t^f, \tag{C.7}$$

where we defined:

$$\tilde{u}_t^f = \log\left(\frac{L_t}{P_t Y_t}\right) + \tilde{y}_t - \log\left[1 - \Theta_L \cdot (1-H_t)^{\frac{\omega-1}{\omega}}\right].$$

[4]Data: https://fred.stlouisfed.org/graph/?g=VLW.

We then construct $\tilde{u}_t^f$ based on available data on loan-to-GDP ratio and filtered GDP with elicited H_t series[5] and then run equation (C.7) as a linear regression, where $(1-\rho_f)\cdot\log(\phi_f)$ will be contained in the constant term. This is equivalent to estimating an AR(1) process.

We obtain an estimate for ρ_f from regression (C.7), which we use with the constant term equal to $(1-\rho_f)\cdot\log(\phi_f)$ to back out a new estimate for ϕ_f. Finally, we obtain an estimate for σ_f as the standard deviation of the residual.

Step 4 Using the new estimate for ϕ_f, repeat the estimation process from **Step 2**. We iterate until the value of estimated ϕ_f converges.

C.4 Estimation of the Policy Room $\frac{R_t^B}{R_t^{J,*}}$

As policy room $\frac{R_t^B}{R_t^{J,*}}$ is not observable, we use the estimated H_t series (either from Version 1 or Version 2) in eliciting $\frac{R_t^B}{R_t^{J,*}}$ series. Rearranging equation (31), we obtain:

$$\frac{R_t^B}{R_t^{J,*}} = \left(\frac{\omega+1}{\omega}\right)\cdot(1-H_t)^{\frac{1}{\omega}}$$

which we can estimate by plugging in the values of estimated H_t. Dividing the previous equation by its steady-state value, we obtain:

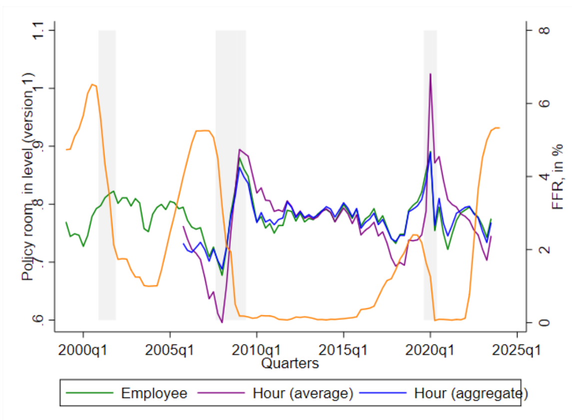
$$\left(\frac{R_t^B}{R_t^{J,*}}\right)\div\left(\frac{R^B}{R^{J,*}}\right) = \left(\frac{1-H_t}{1-H}\right)^{\frac{1}{\omega}}$$

which in log-deviations becomes:

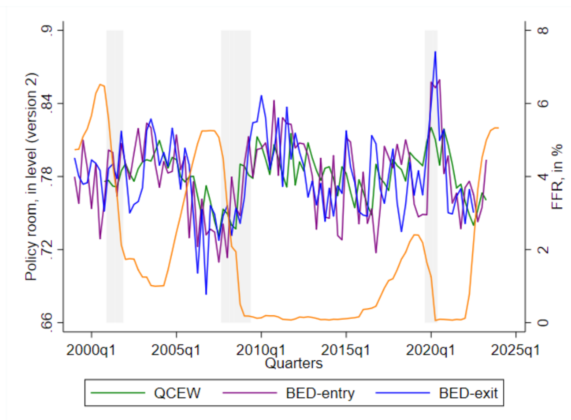
$$\widehat{r_t^B - r_t^{J,*}} = \frac{1}{\omega}\cdot\log\widehat{(1-H_t)} \tag{C.8}$$

which is the expression that we use as a proxy for the policy room in our empirical analysis. Figure C.5 depicts the policy room series based on Version 1 and Version 2, respectively. We observe that the policy room tends to spike during the recession where the monetary policy rate tends to be low and around zero (i.e., zero lower bound).

[5] Note that in eliciting H_t series, we need the value of H, the steady state level of H_t, which relies on the guessed value of ϕ_f.



(a) Policy room: Version 1



(b) Policy room: Version 2

Figure C.5: Policy room: Version 1 and Version 2

Notes: For Version 1, the three series are estimated based on three sources of information on the employment N. Employee: number of employees. Hour (average): average weekly hour. Hour (aggregate): aggregate weekly hour, in thousands. The yellow solid line is the federal funds rates. For Version 2, the three time series are estimated based on three sources of information on the operating firms M. The green line is based on the number of establishments in QECW database. The purple (blue) line is the number of establishments based on firm entry (exit) information in BED. The yellow line is the federal funds rates. The equation is

$$\text{Number of establishment} = \frac{\text{number of establishments with employment gains (loss)}}{\text{percentage of establishments with employment gains (loss)}}.$$

D Additional Tables and Figures

D.1 Section 3.2

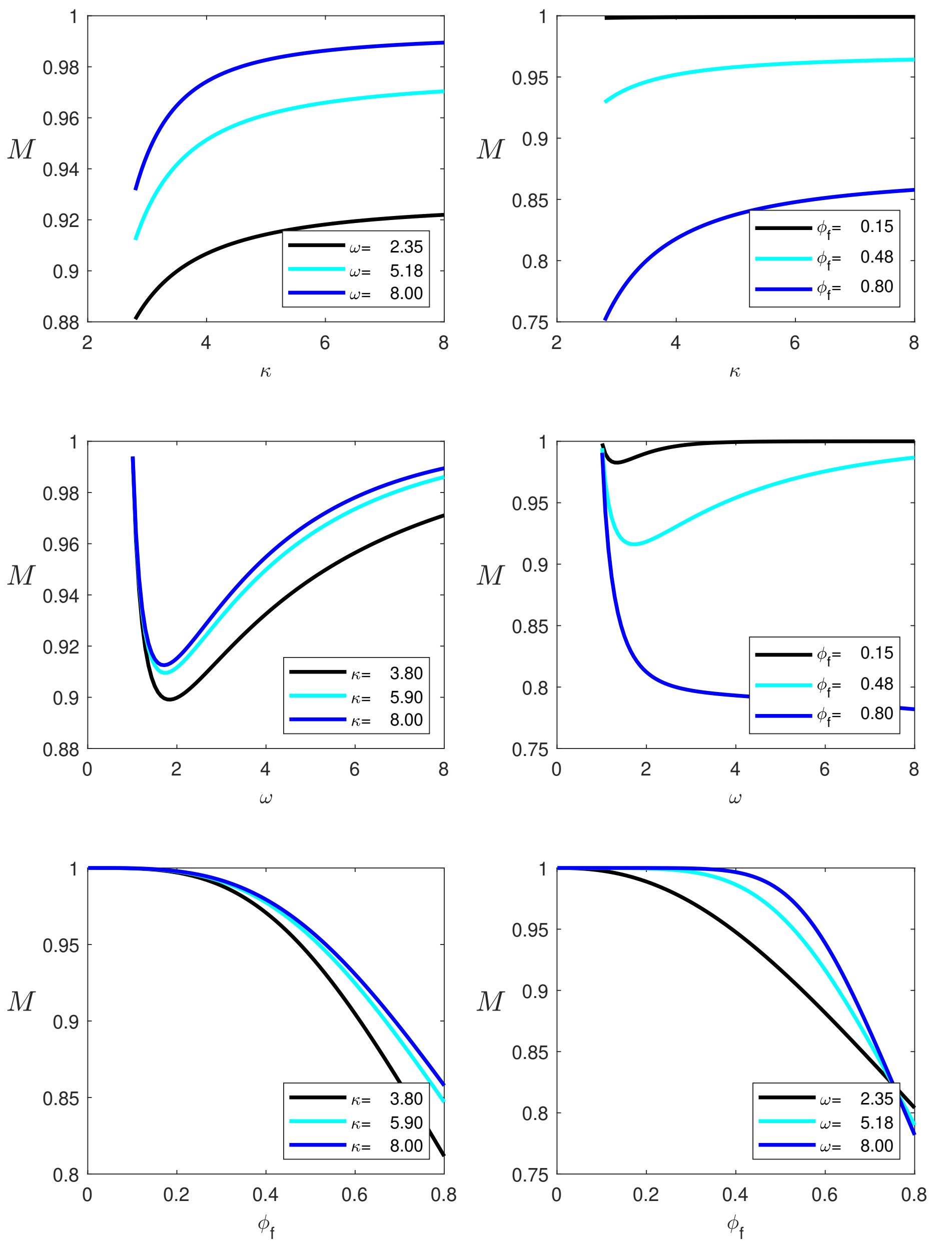


Figure D.6: Comparative Statics: M.

Notes: This figure displays how variations in other structural parameters affect the relation between M and the structural parameters.

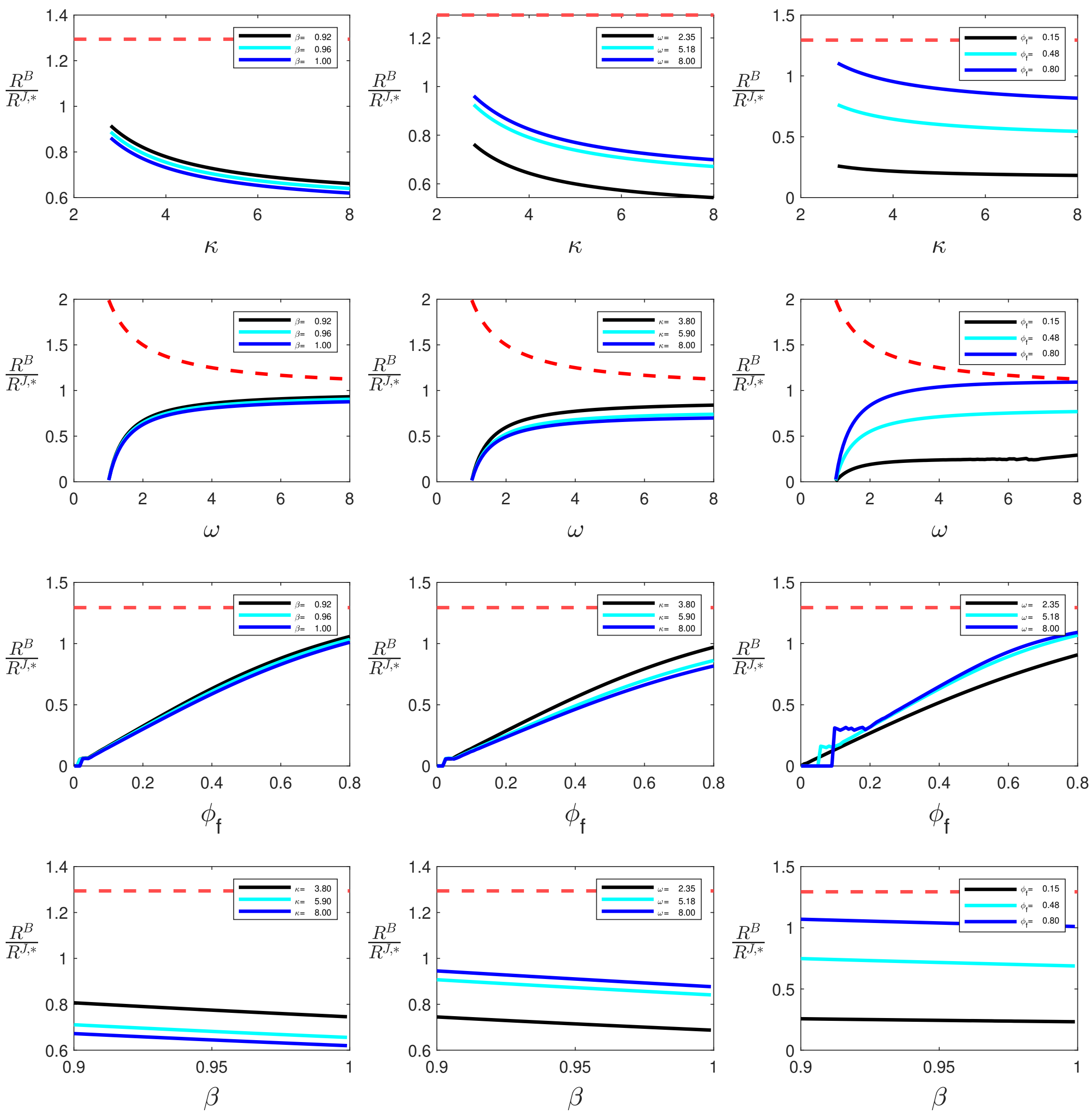


Figure D.7: Comparative Statics: Policy Room.

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between the policy room and the parameters.

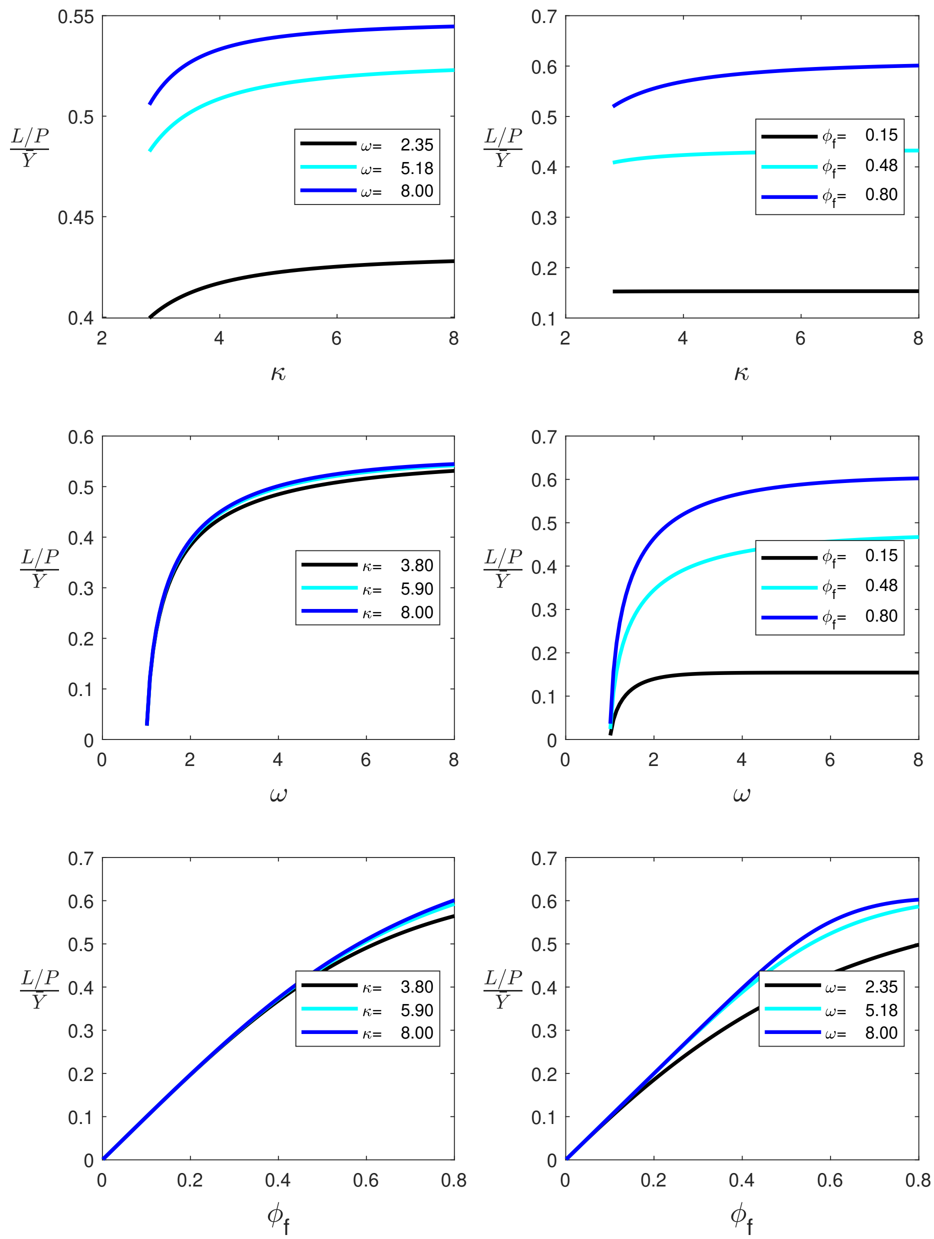


Figure D.8: Comparative Statics: Loan-to-output ratio.

Notes: This figure display how κ, ω, and ϕ_f affect the relationship between $\frac{L/P}{Y}$ and the parameters.

D.2 Section 4.1

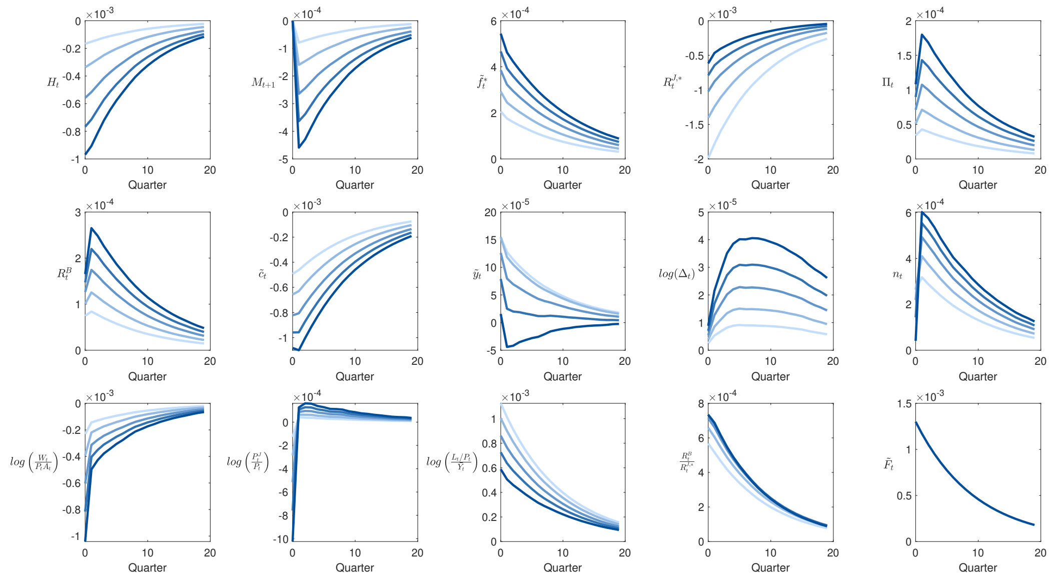


Figure D.9: Impulse response functions to fixed cost shock.

Notes: The figures display the deviation for 1 positive standard deviation (0.0013) in $u_{f,t}$, the fixed cost shock. The autoregressive coefficient is 0.6. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The variables below are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$ (net interest rate). The rest of the variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ is the price dispersion for the downstream products. $W_t/(P_tA_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

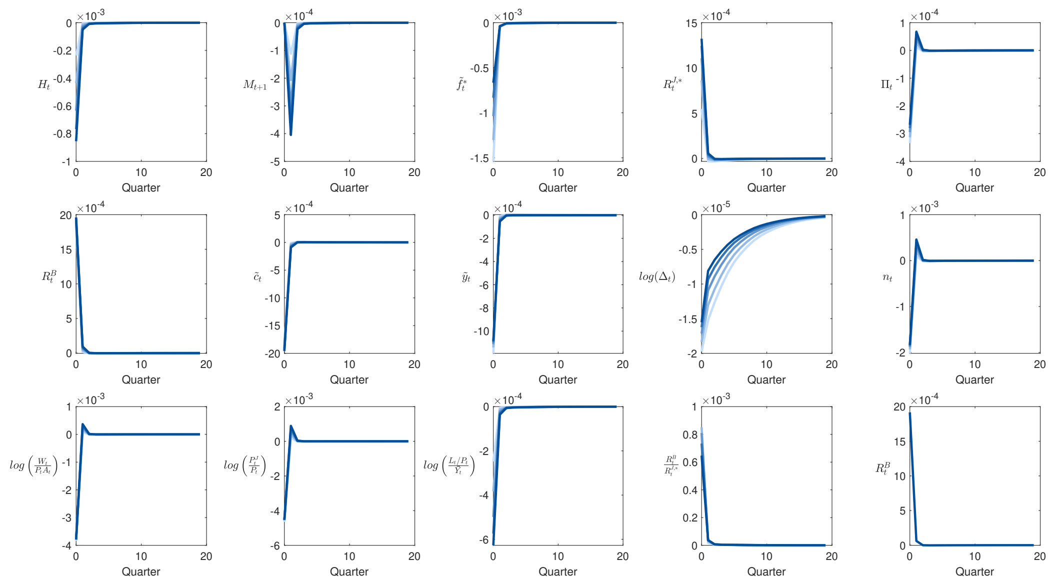


Figure D.10: Impulse response functions to monetary policy shock.

Notes: The figures display the deviation for 1 positive standard deviation (0.0025) in $\epsilon_{r,t}$, the monetary policy shock. The gradient blue lines denote the responses under calibrations with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The variables are plotted in deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$ (net interest rate). The rest of the variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ is the price dispersion for the downstream products.$W_t/(P_tA_t)$ is the real wage. P^J_t/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

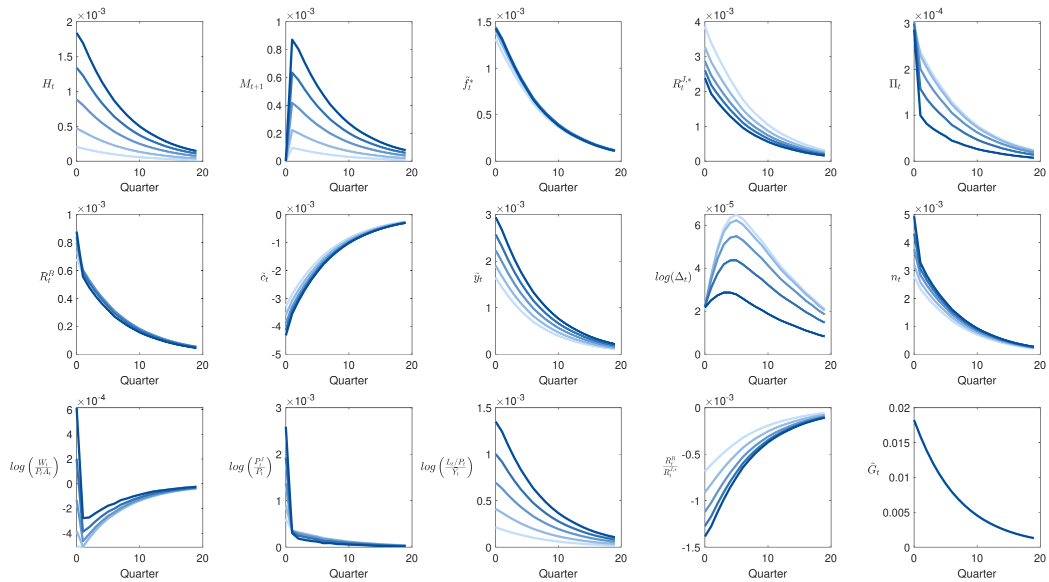


Figure D.11: Impulse response functions to government spending shock.

Notes: The figures display the deviation for 1 positive standard deviation (0.016) in $u_{g,t}$ which denotes the government spending shock. The autoregressive coefficient is 0.97. The gradient blue lines denote the responses under calibration with varying ϕ_f. From the light blue to the dark blue, ϕ_f are 0.35, 0.45, 0.5547 (benchmark), 0.65, and 0.75. The variables below are plotted in level deviations from their steady states: H, M, R^B, Π, and $R^{J,*}$ (net interest rate). The rest of the variables are plotted in log deviations from their steady states (in lower case letters or with a log). Δ is the price dispersion for the downstream products. $W_t/(P_tA_t)$ is the real wage. P_t^J/P_t measures the aggregate price for the upstream products or the input price for the downstream firms.

D.3 Section 4.3

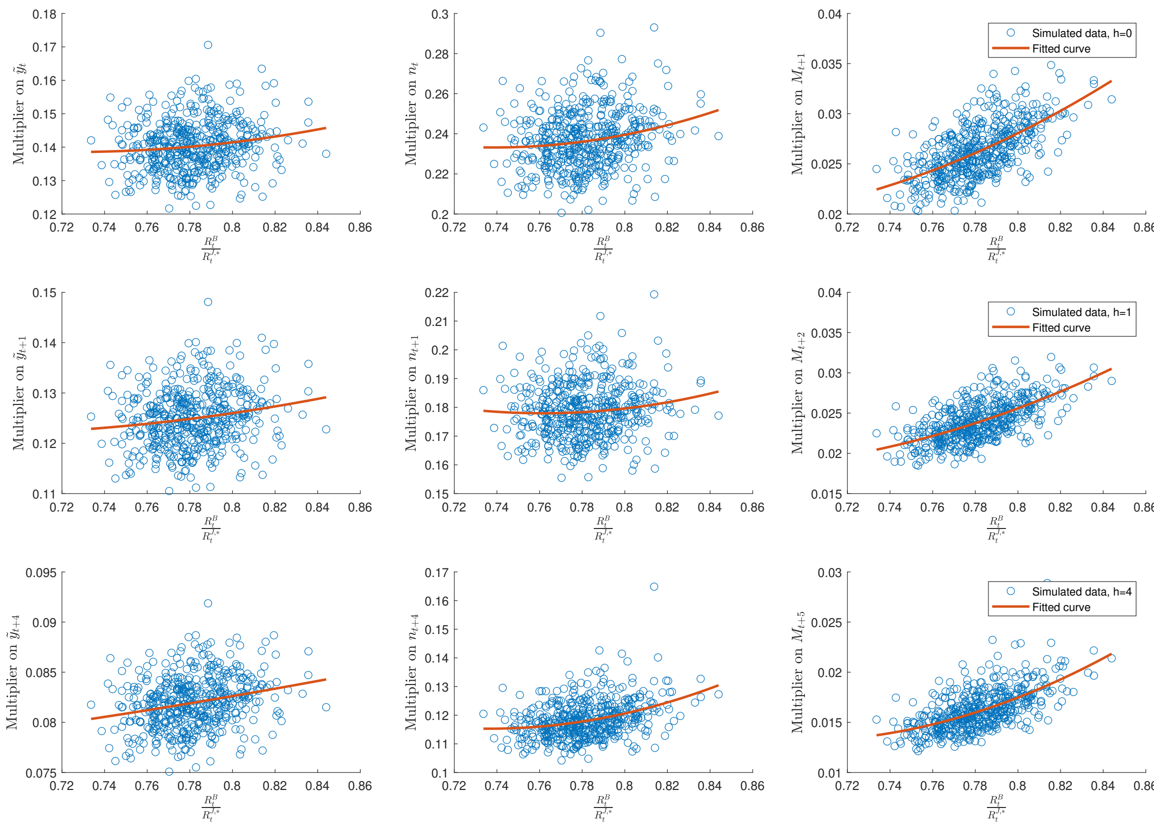


Figure D.12: Scatter plot between policy room and government spending multipliers.

Notes: Figures plot the relationship between policy room and government spending multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t+1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

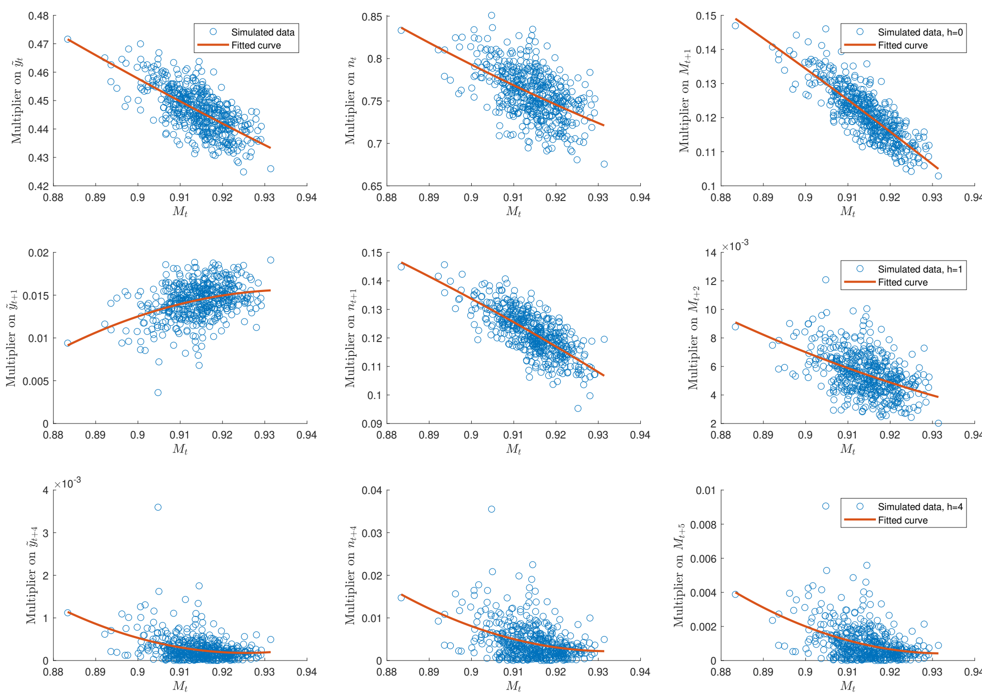


Figure D.13: Scatter plot between the mass of firms and monetary policy multipliers.

Notes: Figures plot the relationship between the current mass of operating firms and monetary policy multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t + 1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

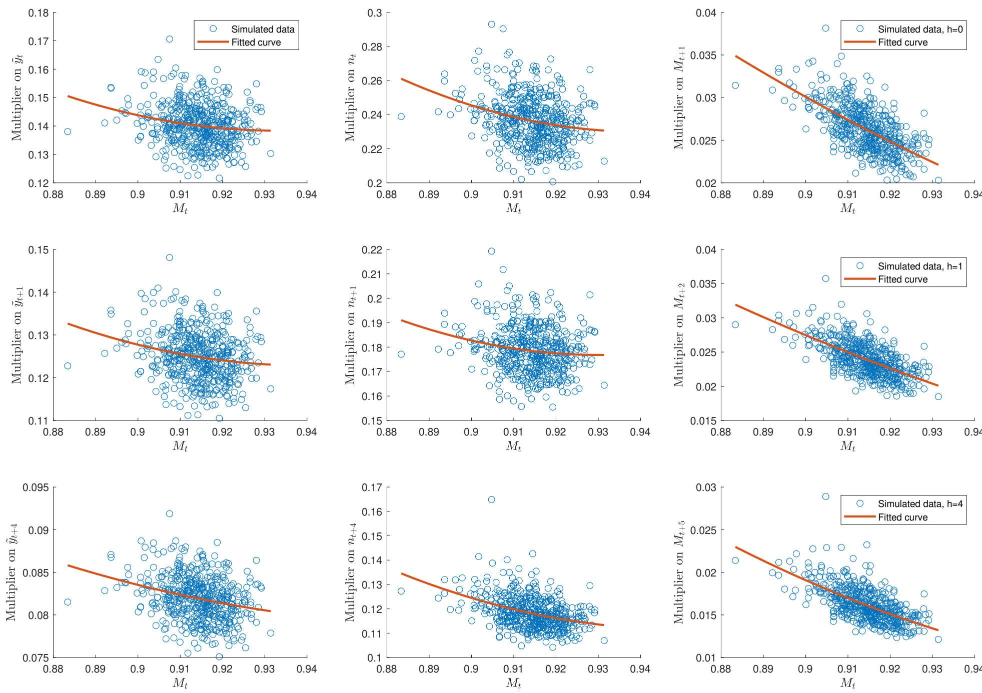


Figure D.14: Scatter plot between the mass of firms and government spending multipliers.

Notes: Figures plot the relationship between the current mass of operating firms and government spending multipliers on output (in logs), labor (in logs), and next period mass of operating firms (in levels). We consider the next period's mass of operating firms since the firms paying the fixed cost at t will operate on the market at $t+1$. Figures in the first to third rows display the contemporaneous multipliers ($h = 0$), multipliers after 1 quarter ($h = 1$), and multipliers after 4 quarters ($h = 4$) correspondingly. The blue circles represent the result from each simulation based on solutions from the third-order perturbation method. The red solid lines are fitted second-order polynomials.

E Robustness in Section 5

E.1 Robustnes: the Policy Room Recovered by Version 1

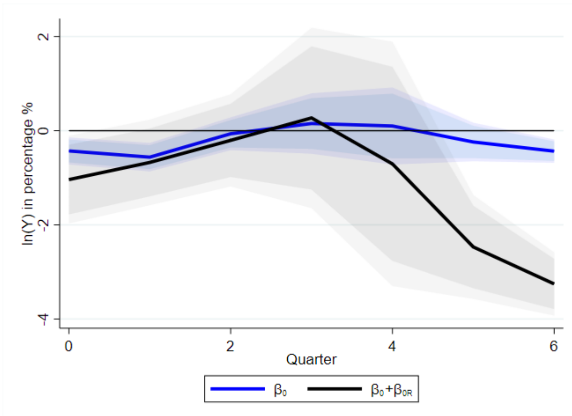
Now, we run our benchmark regression

$$\tilde{y}_{t,t+h} = \sum_{q=1}^{Q} \tilde{y}_{t-q} + \sum_{m=0}^{M} \beta_{0,m}^{(h)} \epsilon_{t-m} + \sum_{n=1}^{N} \beta_{R,n}^{(h)} \widehat{r_{t-m}^{B} - r_{t-m}^{J*}} + \sum_{p=0}^{Q} \beta_{0R,p}^{(h)} \epsilon_{t-p} \times \widehat{r_{t-p-1}^{B} - r_{t-p-1}^{J*}} + u_{t+h|t}$$

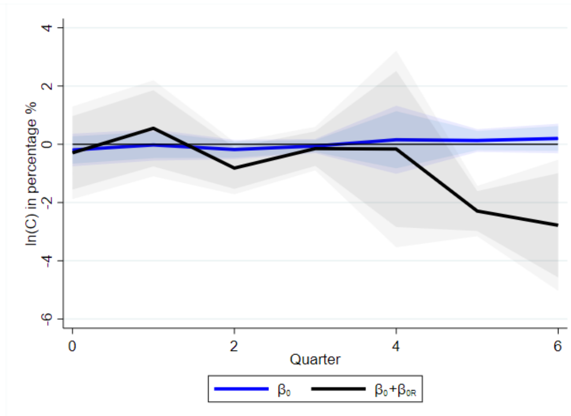
based on the policy room measure recovered by Version 1 in Appendix C.1 and Appendix C.4, based on the total number of employees from CES National Databases in the Bureau of Labor Statistics (BLS). In this case, we use monetary shock series from either Wieland and Yang (2020) or Acosta (2023), who both extended the shock series of Romer and Romer (2004).

Figures E.15 (with Wieland and Yang (2020) monetary policy shock series) and E.16 (with Acosta (2023) shock series) display the impulse response functions of output, consumption, and unemployment to monetary policy shocks and the interaction of monetary policy shocks with policy room deviation constructed from Version 1 (with the employment measured by the number of employees from BLS). Overall results are similar, even if they become less significant with Acosta (2023) and the policy room constructed with Version 1. As our model lacks physical capital, using the formula for labor demand (i.e., equation (25)) to recover the satiation measure H_t and the policy room $\frac{R_t^B}{R_t^{J,*}}$ might overemphasize the role of the policy room in driving labor demand fluctuations, lowering the significance of the results.

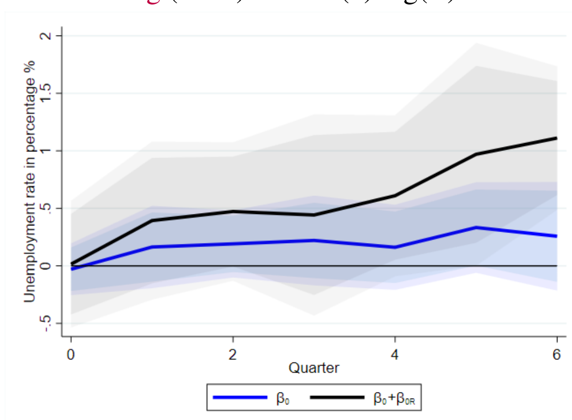
With Additional Controls We add more controls to our benchmark regression with the policy room measure recovered from Version 1 and test the robustness of our results. The controls are current and four lags of federal funds rates, four lags of oil price growth rate, four lags of long-term interest rate, four lags of consumption growth rate, four lags of GDP deflator, four lags of shadow federal funds rate from Wu and Xia (2016). Figure E.17 shows no to little difference from Figure E.16 where no additional control is added. The additional controls are insignificant and do not affect our results.



(a) log(Y) with Wieland and Yang (2020)



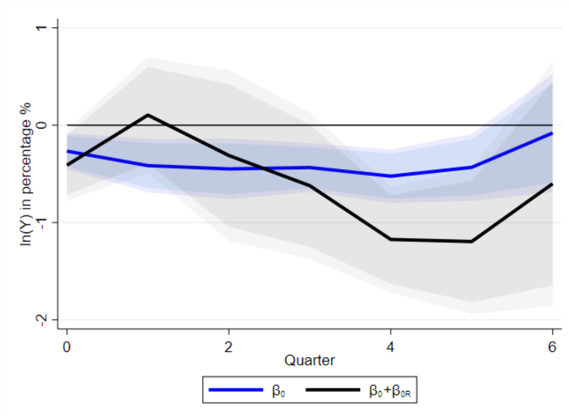
(b) log(C) with Wieland and Yang (2020)



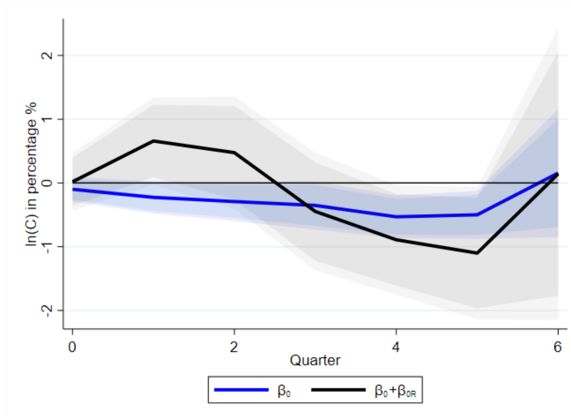
(c) Unemployment rate with Wieland and Yang (2020)

Figure E.15: Local projection: with policy room from Version 1

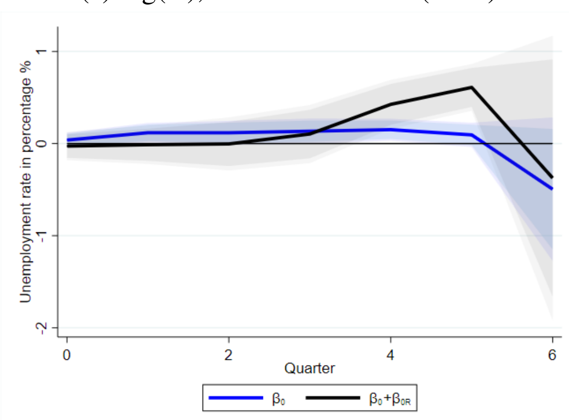
Notes: The impulse response functions are based on the benchmark regression with monetary policy shocks from Wieland and Yang (2020), which controls for current and four lags of federal funds rate.



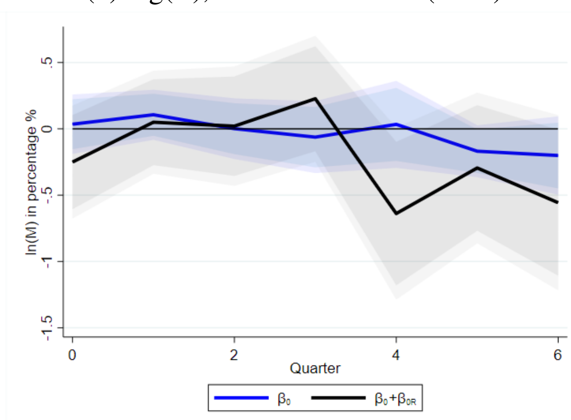
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



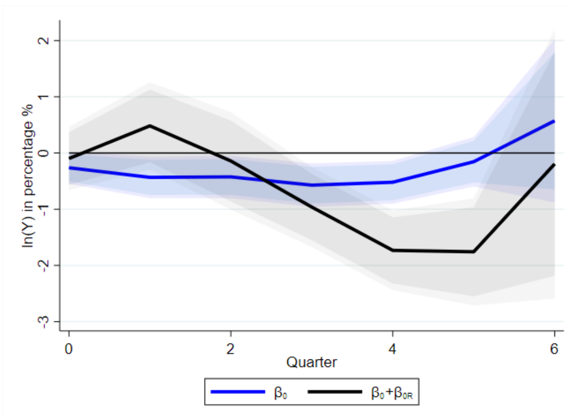
(c) Unemployment rate, in % with Acosta (2023)



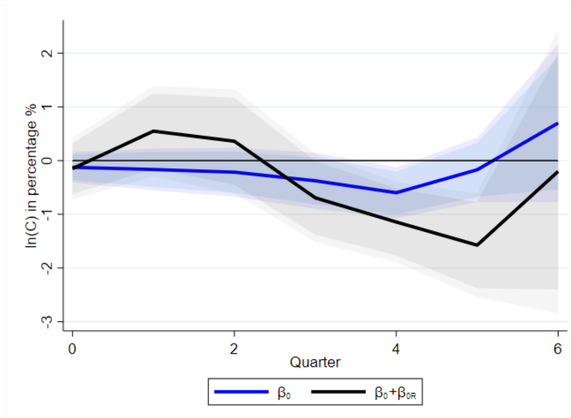
(d) log(M), in % with Acosta (2023)

Figure E.16: Local projection: with policy room from Version 1

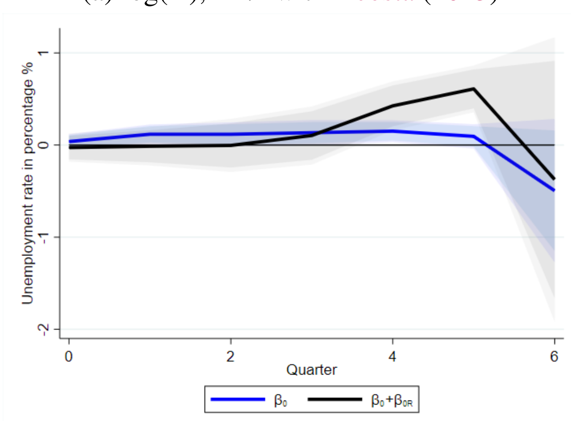
Notes: The impulse response functions are based on the benchmark regression with monetary policy shocks from Acosta (2023), which controls for current and four lags of federal funds rate.



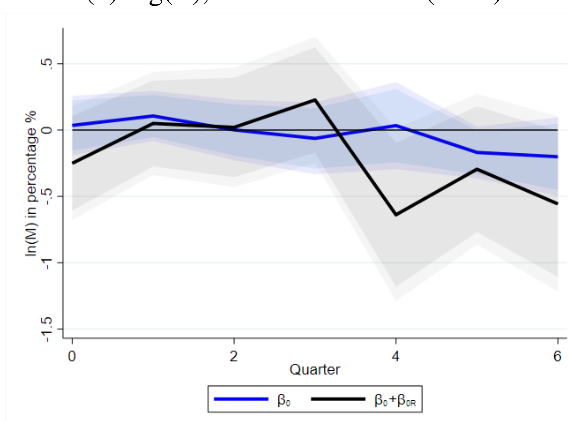
(a) log(Y), in % with Acosta (2023)



(b) log(C), in % with Acosta (2023)



(c) Unemployment rate, in % with Acosta (2023)

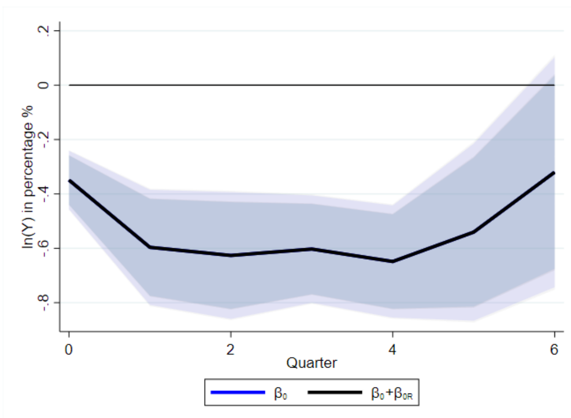


(d) log(M), in % with Acosta (2023)

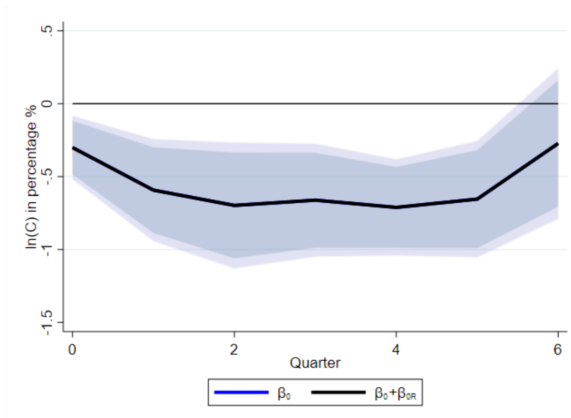
Figure E.17: Local projection: with policy room from Version 1 and additional controls

Notes: The impulse responses functions are for the local projection with the following additional controls: current and four lags of federal funds rates, four lags of oil price growth rate, four lags of long-term interest rate, four lags of consumption growth rate, four lags of GDP deflator, four lags of shadow federal funds rate from Wu and Xia (2016).

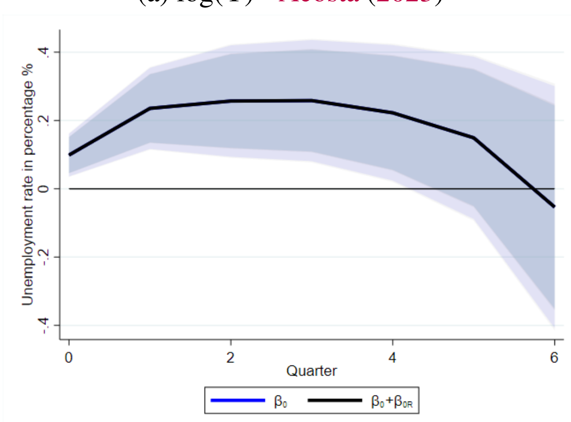
Without Interaction We test the effectiveness of monetary shocks without introducing the interaction term with the policy room, i.e., $\beta_{0R,p}^{(h)} = 0$. Figures E.19 (with Wieland and Yang (2020) shocks) and E.18 (with Acosta (2023) shocks) illustrate the impulse response functions of log output, log private consumption and the unemployment rate to a unit of contractionary monetary policy shocks following the method proposed by Romer and Romer (2004). We observe that monetary policy has mostly significant effects on output, consumption, unemployment rate, and firm entry.



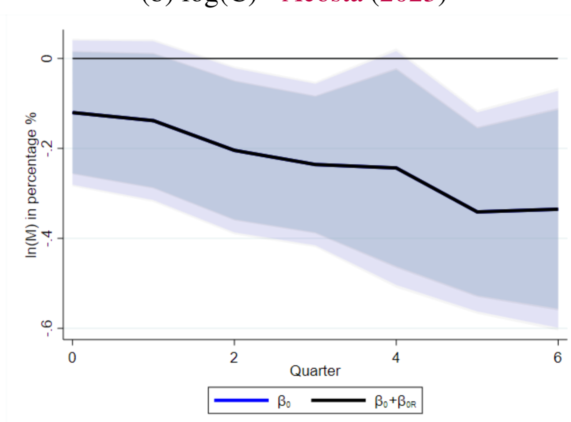
(a) log(Y) - Acosta (2023)



(b) log(C) - Acosta (2023)



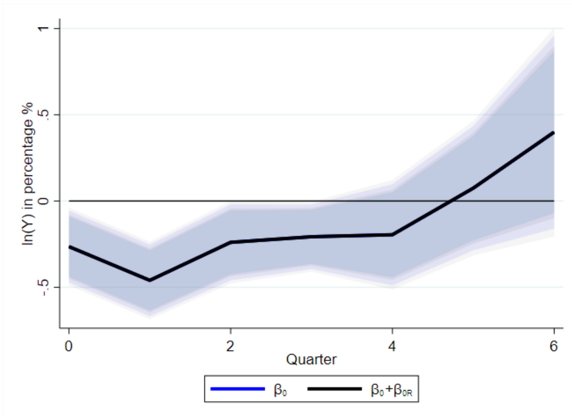
(c) Unemployment rate - Acosta (2023)



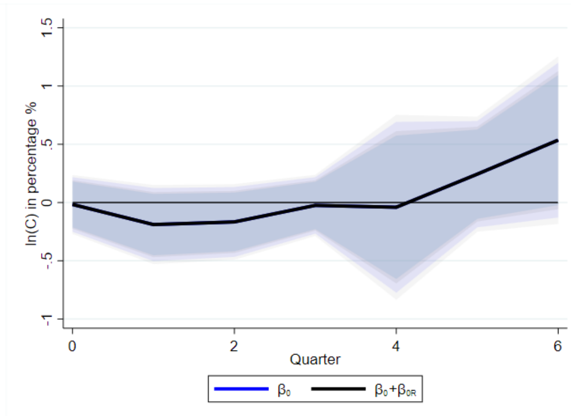
(d) log(M) - Acosta (2023)

Figure E.18: Local projection: without interaction

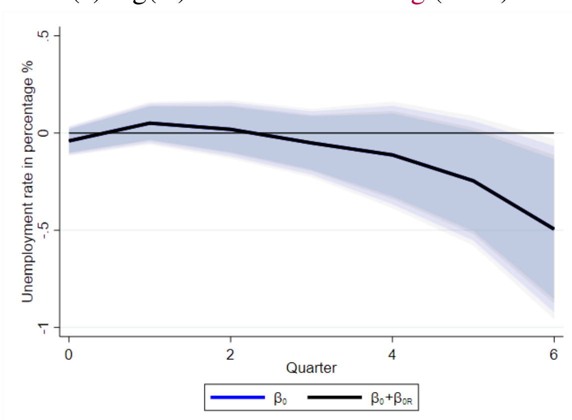
Notes: The impulse response functions are based on the benchmark regression without the interaction term, with monetary policy shocks from Acosta (2023). Controls for current and four lags of federal funds rate.



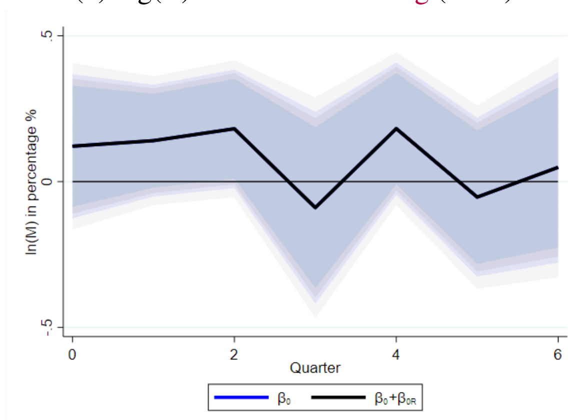
(a) log(Y) - Wieland and Yang (2020)



(b) log(C) - Wieland and Yang (2020)



(c) Unemployment rate - Wieland and Yang (2020)



(d) log(M) - Wieland and Yang (2020)

Figure E.19: Local projection: without interaction

Notes: The impulse response functions are based on the benchmark regression without the interaction term, with monetary policy shocks from Wieland and Yang (2020). Controls for current and four lags of federal funds rate.

E.2 Robustness: the Number of Lags

We summarize the results with the number of lags different from 4, our benchmark number in Section 5 and Appendix E.1, as follows:

Change 1 Number of lags being 2 instead of 4

(a) Impulse response functions with shocks of Wieland and Yang (2020) become smoother with less spiky (also wider and insignificant) confidence bands.

(b) Impulse response functions with shocks of Acosta (2023) are robust when the number of lags in controls changes from 4 to 2, making the results look slightly more significant (the initial positive responses of β_{OR} become smaller and insignificant for output and consumption, thus the later negative responses stand out more significant for output and consumption. Also, the confidence bands for unemployment become narrower for Version-1-based policy room.

(c) Impulse response functions with shocks of Acosta (2023) with Version-2-based policy room (our benchmark result in Section 5) do not change much.

Change 2 Number of lags being 6 instead of 4

(a) Controlling more lags makes the confidence bands of the results with shocks of Wieland and Yang (2020) narrower.

(b) With the number of lags being 6 and the policy room recovered by Version 1, the results become worse with shocks of Acosta (2023): initial positive responses in β_{OR} become more significant and later negative responses are less significant for output and consumption, with larger confidence bands for responses in unemployment.

(c) Impulse response functions with shocks of Acosta (2023) with Version-2-based policy room (our benchmark result in Section 5) do not change much.[6]

[6]The results with too many controls display more spikiness with not-well-behaved confidence intervals though.

F Limiting Case with $\omega \to \infty$

When $\omega \to +\infty$, the Pareto distribution $H(F_{m,t})$ of the fixed costs collapse to its mean, F_t. In this scenario, it is trivial to see that $P^J_{m,t} = P^J_t$. For P^J_t, we plug equation (A.5) into equation (A.3), and obtain

$$\frac{P^J_t}{P_t} = \begin{cases} \Theta_1^{-\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \\ \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1}\left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right) \Pi_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\right]} \right)^{\frac{[\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)](\alpha+\sigma(1-\alpha))}{(\sigma-1)^2\alpha}} \\ \underline{\text{if } R^J_t > R^{J,*}_t}, \\ \\ \Theta_1^{-\left(\frac{\alpha+\sigma(1-\alpha)}{(\sigma-1)\alpha}\right)} \left(\frac{W_t}{P_t A_t}\right) \cdot \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1-\alpha}{\alpha}} \quad \underline{\text{if } R^J_t \le R^{J,*}_t}. \end{cases} \tag{F.1}$$

Plugging (F.1) into (A.5), we can obtain

$$\Xi_t = \begin{cases} \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \\ \cdot \left(\frac{R^J_{t-1} F_{t-1}}{\Theta_2 E_{t-1}\left[\xi_t \left(\frac{P^J_t}{P_t}\right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{W_t}{P_t A_t}\right)^{-\frac{(\sigma-1)\alpha}{\alpha+\sigma(1-\alpha)}} \left(\frac{\kappa-1}{\kappa} A_t\right)^{\frac{(\sigma-1)(1-\alpha)}{\alpha+\sigma(1-\alpha)}} \Pi_t (Y_t \Delta_t)^{\frac{1}{\alpha+\sigma(1-\alpha)}}\right]} \right)^{\frac{\sigma}{\sigma-1}\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]-(\sigma-1)}{(\sigma-1)\alpha}\right)} \\ \underline{\text{if } R^J_t > R^{J,*}_t}, \\ \\ \Theta_5 \cdot \left(\frac{W_t}{P_t A_t}\right) \left[\left(\frac{\kappa-1}{\kappa}\right) A_t\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}} \cdot P_t \left(\frac{Y_t \Delta_t}{A_t}\right)^{\frac{1}{\alpha}} \quad \underline{\text{if } R^J_t \le R^{J,*}_t}, \end{cases} \tag{F.2}$$

where we define

$$\Theta_5 = \Theta_1^{-\left(\frac{\sigma}{(\sigma-1)\alpha}\right)} \Theta_2 \left(\frac{\kappa-1}{\kappa}\right)^{\frac{\alpha(1-\sigma)-1}{\alpha+\sigma(1-\alpha)}}.$$

Now that $M_t = M_{m,t}$, $L_t = L_{m,t}$, $R^{J,*}_t = R^{J,*}_{m,t}$ and $\varphi^*_t = \varphi^*_{m,t}$, we can substitute (F.2) into (14), (15), (16), and (17) to obtain following analytical expressions:

$$R_t^{J,*} = \Theta_5 \cdot E_t \left[\xi_{t+1} \left(\frac{\kappa - 1}{\kappa} A_{t+1} \right)^{\frac{\sigma}{\alpha+\sigma(1-\alpha)}} \left(\frac{w_{t+1}}{P_{t+1} A_{t+1}} \right) \frac{\Pi_{t+1}}{F_t} \left(\frac{Y_{t+1} \Delta_{t+1}}{A_{t+1}} \right)^{\frac{1}{\alpha}} \right],$$

and

$$\varphi_t^* = \left(\frac{R_t^J}{R_t^{J,*}} \right)^{\left(\frac{\alpha+\sigma(1-\alpha)}{\sigma-1} \right)} \left[\left(\frac{\kappa - 1}{\kappa} \right) A_{t+1} \right], \tag{F.3}$$

$$M_{t+1} = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1} \right)} & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ 1 & \underline{\text{if } R_t^J \leq R_t^{J,*}}, \end{cases} \tag{F.4}$$

$$L_t = \begin{cases} \left(\frac{R_t^J}{R_t^{J,*}} \right)^{-\left(\frac{\kappa[\alpha+\sigma(1-\alpha)]}{\sigma-1} \right)} \cdot F_t & \underline{\text{if } R_t^J > R_t^{J,*}}, \\ F_t & \underline{\text{if } R_t^J \leq R_t^{J,*}}. \end{cases} \tag{F.5}$$

We observe: if $R_t^J \leq R_t^{J,*}$, where $R_t^{J,*}$ is defined in (22), all firms are satiated and the loan amount made to firms is equal to F_t, the fixed cost that operating firms need to pay one period in advance.