Diagnostic Expectations in Housing Price Dynamics

by QI Zhenghua

Department of Economics

The Hong Kong University of Science and Technology

Abstract

Using the Survey of Consumer Expectations, I find predictability of forecast error on forecast in housing price growth rate. Overestimation is followed by disappointment in housing market. To resolve the predictability without rational expectation (RE), I introduce the diagnostic expectation (DE) into a two-agent New Keynesian framework to understand the role played by over-optimism induced by DE in affecting housing price dynamics and the business cycles. Firstly, the DE significantly outperforms RE in affecting the responses of housing price, consumption, output and other macroeconomic variables to TFP shock in magnitude and persistency. The DE plays significant role in affecting both the extensive margin and intensive margin of housing market. Secondly, the DE is able to generate positive residential housing value share response that is consistent with data. The main mechanism comes from the strengthened income effect and consumption smoothing under overreaction. Thirdly, the monetary policy rule matters in influencing the mechanisms generated by DE and this laid foundation for future discussion on policy implication.

Chapter 1

Introduction

Housing's multi-role as the consumption good, production capital, and collateralizable asset connects asset pricing and agents' decision-making in the real economy. Moreover, the fluctuations in the housing market contribute to the business cycles through household balance sheets and financial systems. The Great Recession has also documented that with credit expansion, the housing market could initiate booms and busts that affect the real economy's stability and welfare tremendously. All these considerations indicate that understanding the source of housing price fluctuations and the interaction between the housing market and the real economy is naturally important.

Structural shocks, like total factor productivity (TFP) shock and monetary policy (MP) shock, contribute substantially to the volatility of housing price dynamics; for example, see Figure 1. TFP shocks and MP shocks in total account for approximately 20 (40) percent of the forecast error variance of housing price at a 2-year (5-year) horizon. While with these structural shocks, the standard models under rational expectation cannot generate the observed housing price fluctuation unless further assumptions on persistency and magnitude are imposed. One way to resolve this gap is to introduce irrational expectations. Among the possible irrational expectations, this paper mainly focuses on the diagnostic expectation (DE) formulated by Bordalo et al. [6]. Researches under DE have been carried on the stock market (Bordalo et al. [9]) and credit market (Bordalo et al. [6], Bordalo et al. [8]) and yet left blank for the housing market. The diagnostic expectation is rooted in the representative heuristic proposed by Kahneman and Tversky [20], which describes how individuals assess probability by representativeness and depart from the standard Bayesian way. The individual will tend to overestimate the likelihood of an event if it is more representative under its belief. For example, a series of recent good news in housing price will make a higher return in housing investment more representative in individuals' memory, leading to an over-optimistic attitude on housing price growth forecasts. Such overestimation could lead to individuals' increasing demand and further push up the housing price. Under this expectation formation process, agents' over-optimistic and over-pessimistic behaviors could explain part of the observed housing price volatilities which will be missing under RE. In this paper, I construct a two-agent New Keynesian model under DE to show how the structural shocks combined with the amplification effect from diagnostic belief can help to explain the volatility in housing prices. This paper also presents the different interactions of housing prices and key macroeconomic variables under DE and RE.

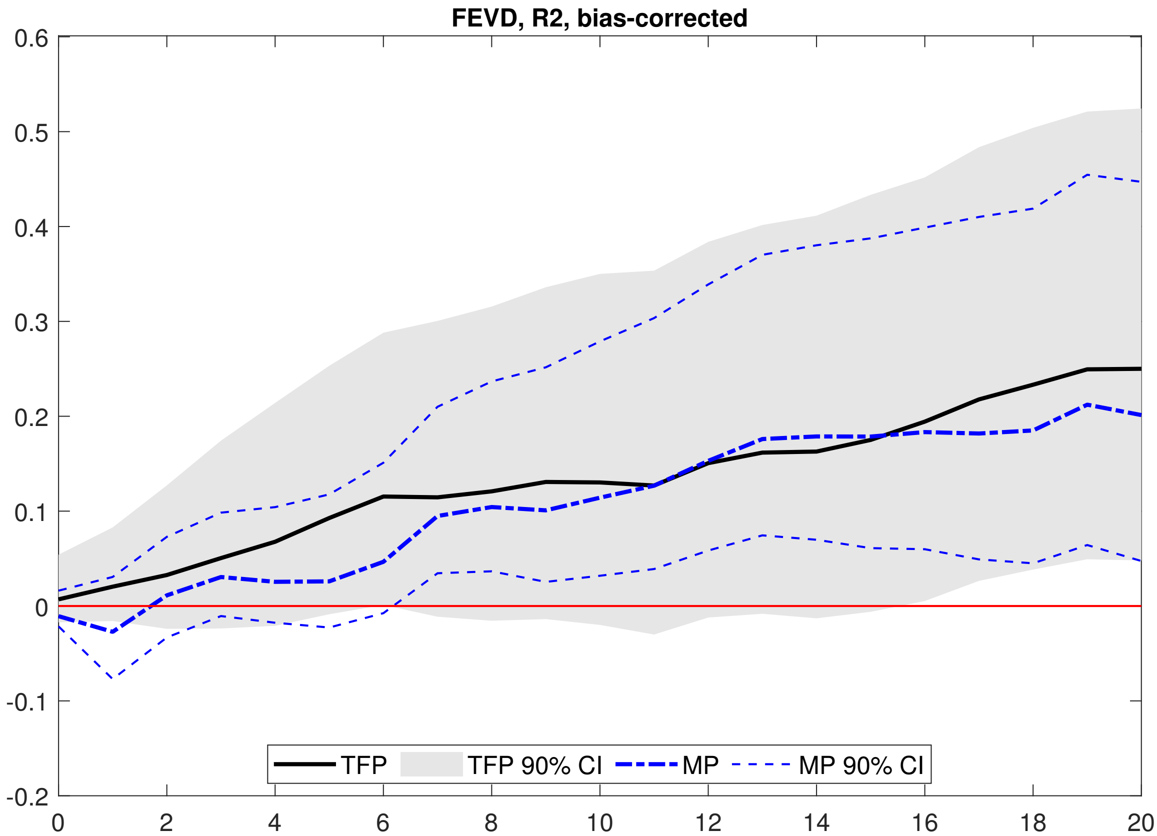


Figure 1: Empirical Forecast Error Variance Decomposition, Real Housing Price

Notes: The forecast error variance decomposition of real housing price is estimated under the local projection framework proposed by Gorodnichenko and Lee [17], more details of estimation can be seen in the paper. The TFP shocks are identified by Fernald [15] and the MP shocks are identified by Romer and Romer [29] and extended by Coibion et al. [11]. The graph display the FEVD with bias-correction with 90% bootstrap confidence interval. The sample covers 1975Q1 - 2008Q4. The unit of y-axis is annualized percentage contribution with the unit of x-axis denotes for quarter.

Specifically, this paper first tests the existence of diagnostic expectation in the housing market by looking at the forecast data and checking the predictability. The forecast errors of housing price growth rate are found to be negatively significantly correlated with the forecasts, revealing deviation from full-information rational expectation assumption and the potential existence of diagnostic expectation. Secondly, this paper incorporates DE within a two-agent New Keynesian framework using the method proposed by Bianchi et al. [4] and finds the amplification and persistency effect of diagnostic expectation. It also turns out that DE can explain the positive response of residential housing value share observed in the data, which cannot be resolved under RE. Overoptimism leads to increasing consumption in both consumption good and housing good even when the marginal product of housing increase when the monetary policy rule does not consider the output gap and not directly affected by DE.

The rest of this paper will be arranged as follows: Section 2 briefly discusses the literature review in two factors affecting housing boom and bust and the application of diagnostic expectations. Section 3 covers the empirical evidence supporting deviation of rational expectation in the housing market. Section 4 discusses the model employed by this paper: starts with a toy model, and then discusses a full model. Section 5 presents the results from the model built and makes comparisons with the empirical findings. Section 6 concludes.

Chapter 2

Literature Review

The research in analyzing the housing market originated a long period ago, and it is impossible to cover even the basic directions in this area's research. While at the same time, there are also pieces of emerging literature growing up on the housing boom-bust cycle with more emphasis on expectation and credit conditions. I only cover the strand of literature that this paper wants to focus on: how the interaction of diagnostic expectation and credit could contribute to the housing price dynamics. So I first review the literature in analyzing some factors affecting housing price dynamics and then review the recent emerging literature in diagnostic expectation.

2.1 Factors Affecting Housing Price Dynamics

Availability of credit and expectations are two key factors commonly believed to drive the fluctuations in housing price dynamics. When the credit is easy to access, the economy will tend to over-borrow and accumulate fragility, experiencing boom and bust according to the shift of the credit conditions. At the same time, the expectation of future returns will affect the agent's decision-making and, therefore, the realized housing price. The sudden shift in the belief of the agents involved in housing transactions, especially the investors, will also contribute to the boom-bust cycle in the housing price dynamics. I summarize the literature on the role of credit conditions and investors' expectations in the following parts.

Role of credit. The interaction of credit conditions and housing price dynamics can work both ways around with generation, amplification, and feedback. Stein [30] first studies the role of down-payment requirements on the fluctuations in housing prices. Housing as collateral will display self-reinforcing propagation and drive further upturn or downturn pattern of housing price. Holding the same sprits, Kiyotaki et al. [22] develop a model where land's role as collateral will affect the sensitivity of housing prices to fundamental shocks. Favilukis et al. [14] construct a general equilibrium model with aggregate business cycle risk and bequest-driven wealth distribution and are able to generate a large boom in housing price after loosening the collateral constraint. Other theoretical works also stress the importance of credit accessibility in explaining the housing boom: Kaplan et al. [21] and Huo and Ríos-Rull [18]. The series of research carried by Mian and Sufi provides fruitful empirical evidence on credit availability and housing price interaction during the Great Recession. Mian and Sufi [26] find the loosen credit constraint, leverage growth, and reliance on credit card have considerable explanatory power in the housing price bust during the recession. Mian and Sufi [27] propose and estimate the feedback effect of leverage on house price. Mian and Sufi [25] connect the housing growth, expansion of debt, and afterward decline in consumption using zip code data during the Great Recession.

In comparing the role of credit and belief, Cox and Ludvigson [12] find that credit plays a vital role in explaining quarterly housing price growth volatility and has predictive power for future growth. The belief matters only contemporaneously.

Role of investor. The investing motive has been found to play a significant role in contributing to the housing price boom before the crisis (for example, Case et al. [10]). Overoptimism spreads across borrowers and mortgage lenders and leads to the accumulation of fragility of the economy. In **?**], even a small fraction of optimistic household's housing transactions at the boom's peak can push up the equilibrium housing price. Besides, the researches by Adelino et al. [1] and Albanesi et al. [3] indicate that prime borrowers and housing investors should be responsible for the cumulated leverage and fragility before the bust. Using the Michigan Survey data, De Stefani [13] directly finds predictability of forecast errors in housing price dynamics. The housing price boom is accompanied by an increase in optimistic investors. At the same time, a positive housing price expectation shock will lead to expansion in leverage originated from investing motivation. Literature in this direction motivates me to consider a two-agent model to discover specifically the role of diagnostic expectation in affecting investors' decision-making.

2.2 Diagnostic Expectation

I only briefly summarize the development of the diagnostic expectation literature. The foundation of diagnostic expectation comes from the representative heuristic proposed by Kahneman and Tversky. They argue that agents' estimation of probability will be affected by representativeness and provide experimental evidence to support their argument. Based on the representative heuristic, Gennaioli and Shleifer [16] consider the judgment bias led by representativeness. Combined with the stereotype formation considered in Bordalo et al. [5], Bordalo et al. [6] formalize the diagnostic expectation and propose the distorted density perceived by the agents with diagnostic expectation as below:

$$h_t^{\theta}(\hat{\omega_{t+1}}) = h(\hat{\omega_{t+1}}|\omega_t = \hat{\omega}_t)\left[\frac{h(\hat{\omega_{t+1}}|\omega_t = \hat{\omega}_t)}{h(\hat{\omega_{t+1}}|\omega_t = b\hat{\omega_{t-1}})}\right]^{\theta}\frac{1}{Z}$$

The variable of interest ω_t follows AR(1) process $\omega_t = b\omega_{t-1} + \varepsilon_t,\ \varepsilon_t \sim i.i.d\ N(0,\sigma^2)$. The $h(\hat{\omega_{t+1}}|\omega_t = \hat{\omega}_t)$ is the true conditional distribution under rational expectation. The $h(\hat{\omega_{t+1}}|\omega_t = b\hat{\omega_{t-1}})$ is the conditional expectation based on past information as reference point. When the state $\hat{\omega}_t$ is more likely to happen relative to the case with past information, this state has the representativeness and leads to an extra weight assigned by the term in the square brackets. The degree of diagnostic expectation is measured by the parameter θ, when $\theta = 0$, the diagnostic expectation reduced to rational expectation. The Z makes the density can be aggregated to 1. Under this density, we can get the expectation expression as below:

$$\mathbb{E}_t^{\theta}(\omega_{t+1}) = \mathbb{E}_t(\omega_{t+1}) + \theta[\mathbb{E}_t(\omega_{t+1}) - \mathbb{E}_{t-1}(\omega_{t+1})] \tag{2.1}$$

The difference between the square bracket in Equation 2.1 comes from the shock. Note here that the reference point is set to be last period's expectation which is not affect by this period shock ε_t. The reference point can also be a combination of past beliefs with various weight assigned. For example, the reference point can be a combination of the past 3-year expectations, indicating the agent has 3-year long memory recall when making predictions for the future realization. The corresponding reference point can be expressed as $\sum_{j=1}^{J}\vartheta_j\mathbb{E}_{t-j}(\omega_{t+1})$.

With this formalization, researches in different market have been carried taking diagnostic expectation into consideration. Bordalo et al.(2019) find negative relationship between realized return with the degree of optimism of corresponding forecasts made by analysts. With this founding, Bordalo et al.(2020) propose the diagnostic bubble model combined with learning and generate the three-phase price path of asset price. Bordalo et al. [7] test the rationality over a series of macroeconomic variables and find over-reaction in individual level and under-reaction in consensus level. Bordalo et al. [8] suggest the real credit cycle theory in which the diagnostic expectation leads to over-investment and accumulation of fragility, generating the boom-bust dynamics at firm and aggregate level and also the counter-cyclical pattern of spreads. Krishnamurthy and Li [23] consider both Bayesian belief updating and diagnostic belief updating in matching crisis cycle while cannot differentiate one from the other.

Chapter 3

Empirical Evidence

In this chapter, this paper first tests the predictability of forecast on forecast errors of housing price growth rate in the U.S. and finds evidence for deviation from the full information rational expectation model. Separate tests on households coming from different geographic groups are also carried in this section. This paper then tests the predictability of forecast revisions on forecast error to further differentiate the cases under irrational expectation and incomplete information rational expectation. The results provide no evidence for the information rigidity under rational expectation case with the existing data. Empirical evidence found in this chapter supports the existence of diagnostic expectation and lays the foundation for model construction in the next chapter.

3.1 Deviation from Full Information Rational Expectation

Data Summary. The forecast data on the housing price index comes from the Survey of Consumer Expectations (SCE) carried out by the Federal Reserve Bank of New York. This survey data provides monthly panel data on consumer expectations about various macroeconomic variables from approximately 1200 household heads in the U.S. coming from different income, education, age, and zip-code groups. The sample spans from June 2013 to May 2021 with 96 observations in time series. Unlike the Survey of Professional Forecasts, this survey data has better representativeness for the belief of normal households. The housing price growth rate forecast error data is constructed as the difference between the expected housing price growth rate from SCE and the realized housing price growth rate. The realized housing price growth rates are calculated separately from three housing price indexes for robustness check: S&P 500

Case-Shiller housing price index, Federal Housing Finance Agency housing price index, and Freddie Mac housing price index. All the results presented in the main sections of this paper are based on the Freddie Mac housing price index because the forecast revision data employed in the next section is constructed from the reports from Freddie Mac.

Under the full information rational expectation, the forecast error should have no predictability power from past data since all the information is considered when making the forecast. I test the predictability of forecast error with the following linear regression model 3.1.

$$FE_{t,t+12}(\Delta HP_{t+12}) \equiv \Delta HP_{t+12} - F_t(\Delta HP_{t+12}) = c_1 + \beta_a F_t(\Delta HP_{t+12}) + controls_t + e_t \quad (3.1)$$

ΔHP_{t+12} denotes for the realized housing price growth rate from time t to $t+12$ (one-year ahead). $F_t(\Delta HP_{t+12})$ is the time t forecast of housing price growth rate for the following year. Thus the forecast error $FE_{t,t+12}(HP)$ is defined to be the difference. The left-hand side of the equation represents the forecast error, while the right-hand side includes forecast and other macroeconomic control variables. c_1 is just the constant term. It is possible that the biased expectation on some macroeconomic variables could also lead to predictability. As a result, two versions of controls are considered here: the controls with only realizations of key macroeconomic variables (Without Expectation version); the controls with both realization of key macroeconomic variables and expectations on inflation, earning and income (With Expectation version). The key macroeconomic variables include the 30-year fixed-rate mortgage rate, total share stock price growth rate, real disposable income growth rate, and inflation rate. The expectation data coming from the SCE data are median one-year ahead expected inflation rate, expected earning growth rate, and expected income growth rate. The results are displayed in Table 2.

Column (1) shows the results for the regression 3.1. Columns (2) to (4) display the results for the same regression for income groups 1 to 3 (below 50k, 50k to 100k, and over 100k) separately. All the estimated coefficients $\hat{\beta}_a$ is strictly significantly smaller than zero, indicating the overestimation in making forecasts. Today's high forecast tends to be accompanied by disappointment tomorrow indicated by smaller realized housing price growth rate relative to forecasted housing price growth rate. The rational expectation is hard to resolve the empirical findings here. One possible explanation of the predictability could be the over-optimism and over-pessimism induced by the representative heuristic. After receiving a series of good news,

Table 1: Tests of the predictability of forecast error

Forecast Error	Median (1)	IC1 (2)	IC2 (3)	IC3 (4)
Without Expectation				
$F_{t,t+12}$	-2.43^{***}	-1.17^{***}	-1.76^{***}	-1.44^{***}
	(0.57)	(0.41)	(0.32)	(0.24)
Adjusted R^2	0.50	0.47	0.51	0.56
Obs	82	82	82	82
With Expectation				
$F_{t,t+12}$	-2.97^{***}	-1.52^{***}	-1.82^{***}	-1.42^{***}
	(0.53)	(0.37)	(0.34)	(0.25)
Adjusted R^2	0.57	0.55	0.50	0.56
Obs	82	82	82	82

Notes: Monthly time series regression: the dependent variable is the actual housing price growth calculate from Freddie Mac housing index over the 12-month period (from time t to $t+12$) minus the predicted housing price growth made by different group for the same period. Median, IC1, IC2 and IC3 denote for median one-year ahead forecast among all participants, among participants from income group 1 (below 50k), income group 2 (from 50k to 100k), and income group 3 (over 100k). The independent variable is the forecast made by the corresponding group at time t for next 12 month's housing price growth. The control variables include mortgage growth rate, inflation, stock price index growth rate and the real disposable growth rate. The Newey-West standard errors are displayed in parentheses. *** denotes for significance under 95% confidence interval.

the agent tends to overestimate the probability that good news will come along in the future, leading them to be overoptimistic and make a higher forecast for housing price growth rate. They will eventually get disappointment since the forecasts are not rational and biased. This extrapolative expectation interpretation is consistent with the studies in other markets, such as the evidence found in firm-level credit cycles (Bordalo et al. 2021), the expectations of macroeconomic variables like Real GDP, unemployment rate made by the professional forecasters (Bordalo et al. 2020).

Besides the predictability, the more interesting finding is that there is no significant difference in the degree of predictability for different income groups. To show this, I introduce dummy variables in the following regression 3.2. The ΔHP_{t+12} term is omitted for short. The $\mathbb{D}_i$ is the dummy variable for income group, for example, $\mathbb{D}_1 = 1$ if the household head comes from the below 50k income group. The estimated $\alpha_{i,2}$ denote for the test on the different degree of irrationality; for example, the estimated $\hat{\alpha_{1,2}}$ denote for the difference between the coefficients in front of regression 3.1 for IC1 and IC2 ($\widehat{\beta_{1,a} - \beta_{2,a}}$).

$$FE_{i,t,t+12} = c_2 + \alpha_{i,1}\mathbb{D}_i + \beta_{i,2}F_{i,t} + \alpha_{i,2}\mathbb{D}_i \times F_{i,t} + \alpha_{i,3}controls_{i,t} + \alpha_{i,4}controls_t + e_{i,t} \quad (3.2)$$

Based on this finding, I propose the framework of investors and savers with same degree of irrational expectation in the model part. The investors correspond to the group of people with higher income, looser credit constraint and better ability to engage in housing investment. Middle-income and high-income households investing in housing constitute this group of people. The savers correspond to the group of people that are not rich enough or risk-aversion to engage into the housing investment. They prefer to save money and invest in risk-free bonds.

Table 2: Tests of the different degree of predictability among income groups

Forecast Error	$i=1$	$i=2$	$i=3$
	(1)	(2)	(3)
$\mathbb{D}_i \times F_{i,t}$	-0.59	-0.35	-0.24
	(0.62)	(0.77)	(0.68)
Obs	164	164	164

Notes: Monthly time series regression: the dummy variables are construct for one-to-one comparison between groups. $\mathbb{D}_1 = 1$ if $i = 1$, corresponding group 1 and same for $\mathbb{D}_2$ and $\mathbb{D}_3$. The time-group specific control variables include the one-year ahead point expectation of inflation, earning growth and income growth rate made by the corresponding group. The time only control variables include mortgage growth rate, inflation, stock price index growth rate and the real disposable growth rate. The Newey-West standard errors are displayed in parentheses.

3.2 Deviation from Full Information

Following Coibion and Gorodnichenko 2015, this paper also examines the predictability of forecast revisions on forecast errors to explore the role of information rigidity. The forecast revision time series is constructed from the monthly and quarterly forecast reports from Freddie Mac, in which the forecasters summarize the actual housing price growth rate in the past four quarters and make forecasts over the housing price in the next four to six quarters. The Survey of Professional Forecasters provides very limited data on housing forecasts and other survey do not have the overlapping responses to calculate the forecast revision. The goodness of the reports from Freddie Mac is that the forecasts have overlaps from which I can construct the forecast revisions manually. The shortcoming of this practice is that this time series is only available from 2014Q1 to 2021Q1, with monthly frequency for the year 2014 and 2015 and quarterly frequency for the rest (37 observations in total). This is not a convincing test but still can give us some flavours on the potential deviation from the rational expectation and further differentiate the case under incomplete information and the case under irrational expectation.

$$\Delta HP_{t+h} - F_t(\Delta HP_{t,t+h}) = c + \beta_b(F_t(\Delta HP_{t,t+h}) - F_{t-1}(\Delta HP_{t,t+h})) + controls_{t-1} + error_t \quad (3.3)$$

As stated in Coibion and Gorodnichenko 2015, the estimated coefficient before forecast revision should be strictly positive under the rational expectation with information frictions. β_b is the coefficient in front of forecast revision denoted by the difference of the time t forecast and time $t-1$ forecast for the same future housing price growth rate. Here the estimated coefficients are all negative for 1-quarter to 3-quarter horizon forecast revision and insignificant, providing no evidence for the role of information rigidity. The empirical evidence in the first section of this chapter reveal deviation from full information rational expectation model while the evidence in the second section provide no further support for deviation from full information. The deviation should come from the rational expectation. Since the data used in this test could be limited, it is possible there is both deviation from full information and from rational expectation at the same time for the housing market. Another argument for the existence of irrational expectation comes from Bordalo, Gennaioli, Ma, and Shleifer (2020). Their results are reconciled by an information learning model under diagnostic expectation. This could be the future direction for the model in this paper.

Table 3: Forecast error on forecast revision, without expectation control

Forecast Revision	h=1	h=2	h=3
	(1)	(2)	(3)
$F_t(\Delta HP_{t+h}) - F_{t-1}(\Delta HP_{t+h})$	-0.97	-0.10	-0.15
	(0.70)	(0.10)	(0.45)
Adjusted R^2	0.13	-0.03	-0.03
Obs	37	36	35

Notes: Monthly and quarterly time series regression: the dependent variables is the forecast error for 1 to 3 quarter ahead housing price growth forecast. The independent variable is the forecast for the next 1 to 3 quarter ahead made by the professionals from Freddie Mac at time t. The Newey-West standard errors are displayed in parentheses

Chapter 4

Model

In this chapter, I first describe a toy model to explain how the predictability of forecast errors in housing price growth rate could be generated under DE and how this irrational belief formation could contribute to the boom and bust cycle in housing price dynamics. Then I introduce a two-agent New Keynesian model combined with the DE using methods proposed by Bianchi et al. [4]. Calibration is taken by matching moments, and simulation results are presented in comparison with those from rational expectation counterparts.

4.1 Demand-Driven Housing Price under Diagnostic Expectation: A Toy Model

There are two types of agents in the economy, housing consumer and housing investor. Housing consumers have constant housing demand denoted by C. Housing investors' demand on housing depends on the probability that the housing price will be higher in next period, denoted by $K\delta_{p,t}$, with $\delta_{p,t} = Prob(p_{t+1} > p_t)$. K is also a constant for simplification. The fundamental housing price will evolve following an AR(1) process, $p_{t+1} = \rho p_t + \varepsilon_{t+1}$, as common knowledge for the agents in the economy. After the realization of time t fundamental housing price shock at the beginning of the time t, the agents are allowed to trade the housing knowing the realized shock. The equilibrium housing price will be settled down at the end of period t. The housing consumer has inelastic demand for housing with respect to prices. The housing investor does not care whether there is a bubble or not: as long as the next period bubble will be larger, they have the motive to hold housing asset even prices may deviate from fundamental values. I assume constant housing supply S and the housing dynamics in this economy is purely driven by housing investors' belief on $\delta_{p,t}$.

Under diagnostic expectation, the expected price increasing probability is denoted as $\delta^{\theta}_{p,t}$. With our belief formation process specified in equation 2.1, the expected next period housing price is $\mathbb{E}^{\theta}_t(p_{t+1}) = \rho p_t + \theta\rho\varepsilon_t$. The predictability of forecast error follows naturally as below, with the absolute value estimated coefficient $\hat{\beta}_a$ increasing with the degree of diagnostic expectation θ:

$$Cov[p_{t+1} - \mathbb{E}^{\theta}_t(p_{t+1}), \mathbb{E}^{\theta}_t(p_{t+1})] = -\theta^2\rho^2\sigma^2 < 0$$

$$\hat{\beta}_a \equiv \frac{Cov[p_{t+1} - \mathbb{E}^{\theta}_t(p_{t+1})}{var[\mathbb{E}^{\theta}_t(p_{t+1})]} = -\frac{\theta^2\rho^2\sigma^2}{\frac{\rho^2}{1-\rho^2}\sigma^2 + \theta^2\rho^2\sigma^2} < 0$$

The probability is $\delta^{\theta}_{p,t} = Prob(\mathbb{E}^{\theta}_t(p_{t+1}) > p_t) = 1 - \Phi[\frac{(1-\rho)p_t - \theta\rho\varepsilon_t}{\sigma}]$. The equilibrium equalizes supply with demand and generates the following:

$$C + K\delta^{\theta}_{p,t} = S$$

$$\frac{S-C}{K} = 1 - \Phi[\frac{(1-\rho)p_t - \theta\rho\varepsilon_t}{\sigma}]$$

At equilibrium, both sides should be constant, thus the realized price process can be derived by normalizing the term within Φ to be 0 and plug back into the fundamental housing price process. The time t housing price is both determined by last period housing price and also the realized shock, following an AR(1) process with different coefficients:

$$p_t = \rho^2 p_{t-1} + (\rho + \theta\rho)\varepsilon_t \tag{4.1}$$

When the belief is rational with $\theta = 0$, the price will stay constant since the probability $\delta_{p,t} = \frac{1}{2}$ for all time t and the investing demand stay constant. When the belief is diagnostic with strictly positive θ, the recent positive realized shock ε_t leads to increase in housing price since the housing investors perceive a higher probability that the housing price will increase tomorrow. For 1 unit positive shock in housing fundamentals, the housing price increase more than 1 unit to achieve equilibrium due to the extra demand from housing investors out of overoptimism. I do not consider the information updating of agents on the law of motion of housing price dynamics for simplification. I restrict this toy model only to explain how the extrapolative belief pushes up the housing price simultaneously generates the predictability of forecast errors. This time-inconsistency issue will be fixed with further assumptions on the agent's belief on future action in the full model section.

4.2 The Full Two-Agent New Keynesian Model

This section combines the diagnostic expectation with a standard infinite-horizon two-agent New Keynesian model to dig the insights in a general equilibrium environment. We already see the time inconsistency issue in the toy model, so before we dive into the details of the full model, I choose a simple way to model belief following Bianchi et al. [4].

4.2.1 A Simple Way to Model Belief

Bianchi et al. [4] formalize two approaches to model the current belief about the future: the naivete and the sophistication. The naivete approach, following the literature by O'Donoghue and Rabin [28] and Akerlof [2], assumes the agent fails in realizing the representativeness heuristic's effect. The agent is confident in his rationality, behaves following the law of motion of the rational agent while does not take into consideration that his expectation is diagnostic. The agent will follow rational policy function but taking diagnostic expected variables as inputs and reference points. This way of modeling belief makes the DE system tractable and can be easily transformed from the RE system with some linear algebra. This paper employs the naivete approach in constructing the DE system. Following the literature by Laibson [24], the sophistication approach assumes the agent is sophisticated enough to realize his expectation is affected by representativeness heuristic and behave under the diagnostic policy function, which requires a fixed point to ensure belief consistency. More conceptual details on memory recall and reference points for sophistication are covered in Bianchi et al. [4]. The naivete approach has the benefit of portability and computational efficiency.

The main idea in employing the naivete approach to characterize the equilibrium system under diagnostic expectation is as follows. More mathematical derivation details on the coefficient matrix can be seen in the Appendix in Bianchi et al. [4].

Step 1: Construct the system under rational expectation, obtain the optimal policy function and recursive law of motion under rational expectation using gensys.

Step 2: Construct the system under diagnostic expectation, with the coefficient matrix calculated from the corresponding matrix under rational expectation, substitute the diagnostic expectation with its definition, and solve the optimal policy function and recursive law of motion under diagnostic expectation.

4.2.2 Households: Patient and Impatient

Following the setting in the toy model, there are two types of representative agents in the economy differentiated by their discount factors. The savers (denote with subscript 1) have a higher discount factor β_1 and are willing to save money. The investors (denote with subscript 2) have a lower discount factor $\beta_2 < \beta_1$ and are willing to borrow money for housing and capital investment. Both type agents enjoy utility from consumption in consumption good, housing good and leisure and make decision on consumption (C_{it}, H_{it}), investment (I_{it}) and involvement in production (N_{it}, $K_{i,t-1}$, $H_{i,t-1}$) to solve the maximization below:

$$\max_{C_{it},N_{it},K_{it},H_{it},\phi_{it},b_t} \mathbb{E}_0^\theta \sum_{t=0}^{\infty} \beta_i^t [lnC_{it} + Jln(\phi_{it}H_{it}) - \chi \frac{N_{it}^{1+\eta}}{1+\eta}]$$

subject to the budget constraint:

$$\begin{aligned} s.t.\ C_{it} + K_{it} + q_t(H_{it} - H_{i,t-1}) + (\mathbb{I}_1 - \mathbb{I}_2)b_t + \frac{\varphi_k}{2}(\frac{I_{it}}{K_{i,t-1}} - \delta)^2 K_{i,t-1} \\ = r_{i,k,t}K_{i,t-1} + r_{i,h,t}(1-\phi_{i,t-1})H_{i,t-1} + w_{it}N_{it} + (1-\delta)K_{i,t-1} + \Pi_t + (\mathbb{I}_1 - \mathbb{I}_2)\frac{R_{t-1}}{1+\pi_t}b_{t-1} \end{aligned} \tag{4.2}$$

The investors as the borrowers are restricted by the credit constraint:

$$b_t \le m_t \mathbb{E}_t^\theta (q_{t+1}\frac{H_{2t}}{R_t}) \tag{4.3}$$

Households take two steps in allocating housing. They first choose the total amount of housing holding as H_t at period t. Then, the total housing holding can be divided into ϕ_{it} proportion of consumption need and $(1-\phi_{it})$ proportion of production need. The $\mathbb{I}_1$ is an indicator function that equals to 1 if the agent is type 1 (saver) and 0 otherwise. The J is the preference parameter on housing consumption. q_t is the housing price that is assumed to be the same for residential and commercial housing. δ denotes the capital depreciation rate and I do not consider the housing depreciation in this case. Returns on production inputs are expressed as r_k, r_h, and w. φ_k represents the magnitude of capital adjustment cost. m_t measures the tightness of credit constraint and its log term follows an AR(1) process.

$$logm_t = \rho_m logm_{t-1} + \varepsilon_{m,t}, \quad \varepsilon_{m,t} \sim iid\, N(0, \sigma_m^2)$$

The only friction in this model comes from the credit constraint, which is far from reality. The variations in m_t could have other possible meanings and can be employed to denote other potential factors that are not explicitly considered here, like housing supply shock, housing preference shock, etc. The credit constraint is assume to be binding around the steady state due to the relationship between the two discount factors. I do not consider the occasionally binding credit constraint in this paper. A more detailed discussion will be covered in the future work.

Solving the problem gives us the Euler equations on the household sides, below I only present the housing demand Euler equation and housing distribution Euler equation:

$$JH_{i,t}^{-1} + \beta_i \mathbb{E}_t^{\theta}[C_{i,t+1}^{-1}(r_{i,h,t+1}(1-\phi_{i,t}) + q_{t+1})] + \mathbb{I}_2 m_t \psi_t \mathbb{E}_t^{\theta} q_{t+1} = C_{i,t}^{-1} q_t \tag{4.4}$$

$$J\phi_{it}^{-1} = \beta_i \mathbb{E}_t^{\theta}[C_{i,t+1}^{-1} r_{i,h,t+1} H_{it}] \tag{4.5}$$

$$C_{2,t}^{-1} = \psi_t R_t + \beta_2 \mathbb{E}_t^{\theta}[C_{2,t+1}^{-1} \frac{R_t}{1+\pi_{t+1}}] \tag{4.6}$$

The total housing demand for the saver will be affected by time t and future expected marginal utility of consumption, expected future housing price, and current marginal utility of housing consumption. The investor has one more extra factor coming from the borrowing constraint that will affect his decision. The tradeoff of housing distribution is between the marginal benefit from residential housing consumption utility and marginal cost from commercial real estate production. The diagnostic expectation will affect the economy through affect the decision making though belief channel.

4.2.3 Monopolistically Competitive Firms

This section motivate the sticky price in the standard way as in the literature. A continuum of monopolistic price-setting intermediate good firms producing Y_{jt}, which will be transferred into a final consumption good $Y_t = \left[\int_0^1 Y_{jt}^{\frac{\zeta-1}{\zeta}} dj\right]^{\frac{\zeta}{\zeta-1}}$, the ζ denotes the elasticity of demand. The production function for each intermediate good producer is Cobb-Douglas constant returns-to-scale technology that take capital, housing and labor as inputs:

$$Y_{jt} = A_t K_{j,t-1}^{\alpha} H_{j,t-1}^{\kappa} N_{j,t}^{1-\alpha-\kappa} \tag{4.7}$$

A_t is the technology process that its log term follows AR(1) process:

$$logA_t = \rho_a logA_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim iid\ N(0, \sigma_a^2)$$

The intermediate firm first solve the cost minimization problem to settle down the input and then solve the profit maximization problem by setting price facing demand function from households. I assume the discount factor of firms equals to β_1. The marginal cost solved in the first step is given by $mc_t = \frac{1}{A_t}(\frac{r_{k,t}}{\alpha})^{\alpha}(\frac{r_{h,t}}{\kappa})^{\kappa}(\frac{w_t}{1-\alpha-\kappa})^{1-\alpha-\kappa}$. The sticky price setting is standard Calvo and only $(1-\omega)$ proportion of firms are allowed to set new price, thus I have the aggregate price as $P_t = (\omega P_{t-1}^{1-\zeta} + (1-\omega)P^{*1-\zeta})^{\frac{1}{1-\zeta}}$ and optimal price setting $\frac{p_{j,t}^*}{P_t} = \frac{\zeta}{\zeta-1}\frac{\mathbb{E}_t[\sum_{i=0}^{\infty}\omega^i\beta_1^i mc_{t+i}(\frac{P_{t+i}}{P_t})^{\zeta}]}{\mathbb{E}_t[\sum_{i=0}^{\infty}\omega^i\beta_1^i(\frac{P_{t+i}}{P_t})^{\zeta-1}]}$. The aggregate supply side is settled down.

4.2.4 Monetary Authority

To consider the diagnostic expectation's effect on central bank policy rule, I consider two versions of Taylor rule in this section. Under the first rule, the central bank only responses to current time inflation besides the inertia in nominal interest rate.

$$\frac{R_t}{R} = (\frac{R_{t-1}}{R})^{a_1}[(\frac{\pi_t}{\pi})^{a_2}]^{1-a_1}e^{\nu_t} \tag{4.8}$$

Under the second rule, the central bank also responses to expected inflation and also the output gap denoted by $X_t \equiv \frac{Y_t}{Y^*}$. I assume the central bank's expectation is also affected by the representative heuristic at this stage and do not consider the case where the central bank is rational and aware of the diagnostic expectations of the agents.

$$\frac{R_t}{R} = (\frac{R_{t-1}}{R})^{a_1}[(\frac{\mathbb{E}_t^{\theta}(\pi_{t+1})}{\pi})^{a_2}X_t^{a_3}]^{1-a_1}e^{\nu_t} \tag{4.9}$$

4.2.5 Market Clearing

Here I have two good markets: consumption good market and housing good market. For the consumption good market, symmetry is assumed across intermediate firms. I assume constant housing supply for the housing good market and do not consider the housing supply shock. The labor market clears by equalizing the labor supply with labor demand. The capital market clears with the law of motion of capital and investment cost. The bond market clears with binding credit constraint by assumption.

$$C_{1t} + I_{1t} + C_{2t} + I_{2t} = \frac{1}{g}[Y_{1t} + Y_{2t}]$$

$$\phi_{it} H_{it} + (1 - \phi_{it}) H_{it} = H_{it}$$

$$K_{it} = I_{it} + (1 - \delta) K_{i,t-1}$$

$$b_t R_t = m \mathbb{E}_t q_{t+1} H_{2t}$$

The above market clearing conditions complete the full model in this section.

4.2.6 Calibration

The parameters considered in this model are partly chosen from the literature and partly estimated by matching moments. The standard parameters β_1, ξ, η, δ, α and ζ are set to be 0.99, 1, 1, 0.025, 0.3 and 10. The probability of fixing price ω is set to be 0.75, indicating the average price duration to be 4 quarters. The coefficients in the Taylor rule ψ_1, ψ_2 and ψ_3 equals 0.8, 1.5 and 0.1 to ensure unique equilibrium in New Keynesian framework.

The β_2 is set to be 0.98 to make its implied investor net return to be twice as big as the interest rate implied by $\beta_1 = 0.99$ for the savers. This is set by assumption. The weight on housing preference J is set to 0.075 to match the residential housing value to output ratio of 160% at the steady state observed from the Flow of Funds account data (Table B.101). The production coefficient over commercial housing κ equals 0.03 to match the steady state commercial housing value to output ratio of 67% observed from the Flow of Funds (Table B.103). $\alpha + \kappa = 0.33$ is consistent with the literature of capital have the share of one-third in production process. The summary of calibrated parameters can be seen in Table 4.

The diagnostic expectation parameter θ is set to be 1. The estimation of diagnostic parameter for various macroeconomic variables is broadly discussed and ranges from 0.5 to 1.4

Table 4: Calibrated Parameter Values

Description	Parameter	Value
Discount factor for saver	β_1	0.99
Discount factor for investor	β_2	0.98
Weight on housing preference	J	0.075
Weight on labor dis-utility	χ	1
Labor supply aversion	η	1
Capital depreciation rate	δ	0.025
Capital share in production	α	0.3
Commercial housing share in production	κ	0.03
Elasticity of demand	ζ	10
Probability of fixing price	ω	0.75
Taylor rule on R_{t-1}	a_1	0.8
Taylor rule on $\mathbb{E}_t(\pi_{t+1})$	a_2	1.5
Taylor rule on output gap	a_3	0.1

in Bordalo, Gennaioli, Ma, and Shleifer (2020). The diagnosticity of around 1 is consistent with the founding by research employing different survey data. For example, Bordalo et al. (2018) found $\theta = 0.9$ from professional forecast data on credit supply; d'Arienzo (2020) found $\theta = 1$ with bond price data.

The parameters including autocorrelation, standard deviation related to shocks and capital adjustment cost are estimated using the method of moments (MoM). I estimate these parameters by minimizing the distance between model-generated and empirical-observed variance-covariance matrix of time series in the order of (Y, C, I, N, P): output, consumption, investment, labor, and housing price. Let ι denotes for the vector of parameters to be estimated and $\Upsilon(\iota)$ denotes for the model-implied variance-covariance matrix. $\hat{\Upsilon}$ is the 5×5 matrix of empirical estimate. The estimation of vector ι, solves

$$\min_{\iota}(\Upsilon(\iota) - \hat{\Upsilon})'\mathbb{W}(\Upsilon(\iota) - \hat{\Upsilon}) \tag{4.10}$$

where the weighting matrix $\mathbb{W}$ is chosen to be an identity matrix with extra weight assigned on variance of housing price. I assume all the disturbances other than TFP shock and monetary policy shock are allocated to be part of shock m_t. Table 5 summarize the estimated coefficients under two type of Taylor rule.

To make the standard deviation closer to reality, I choose the starting values for the MoM to be consistent with the standard deviation of the identified shock time series. The TFP shocks are identified by Fernald [15] and the MP shocks are identified by Romer and Romer [29]

Table 5: Estimated Parameter Values

Description	Parameter	Taylor Rule 1	Taylor Rule 2
Autocorrelation of $logA_t$	ρ_a	0.8797	0.9312
Autocorrelation of $logm_t$	ρ_m	0.8483	0.9893
Autocorrelation of $log\nu_t$	ρ_v	0.6427	0.5213
std of $logA_t$	σ_a	0.0085	0.0081
std of $logm_t$	σ_m	0.0476	0.0574
std of $log\nu_t$	σ_v	0.0013	0.0014
capital adjustment cost	φ	5.2468	5.5275

and extended by Coibion et al. [11]. The comparisons of the estimated standard deviation of identified shocks and the estimation results are displayed in Table 6. The generated IRFs are also shown in this table. The IRFs from empirically identified shocks are estimated using local projections with the following equation with $M = Q = 4$:

$$HP_{t+h} - HP_{t-1} = c^{(h)} + \sum_{m=0}^{M} \beta_m^{(h)} x_{t-m} + \sum_{q=1}^{Q} \phi_q^{(h)} \Delta HP_{t-q} + u_{t+h|t-1} \tag{4.11}$$

The IRFs from the model are simulated with the estimated parameters from the MoM.

Table 6: Shocks Comparison

Shock	Value	IRF
Empirical Identified Shocks		
TFP shock σ_a	0.008 (3.57)	5 to 10%
Monetary policy shock σ_v	0.0014 (0.57)	-5 to -10%
Taylor Rule 1		
TFP shock σ_a	0.0085 (2.6)	3 %
Monetary policy shock σ_v	0.0013 (0.52)	-3%
Taylor Rule 2		
TFP shock σ_a	0.0081 (2.84)	3.5 %
Monetary policy shock σ_v	0.0014 (0.57)	-3.5%

Notes: The standard deviation is calculated from quarterly data and shown in real term. The number in the parentheses are annualized percentage standard deviation. The IRFs are also the annualized percentage response of housing price to the corresponding shocks.

Table 7: Variance Comparison

Variance	Empirical	RE	DE1	DE11
Δq	30.2369	3.1805	7.3779	6.7931
	(30.2369)	(6.1587)	(14.5479)	(13.4762

Notes: The table covers the empirical and simulated variance of annualized percentage housing price growth rate under RE, DE1, and DE11. The numbers in the parentheses are the model-simulated variance under Taylor rule 2.

Finally, I also compare the empirical variance of housing price growth rate and that implied from my model under different versions of belief in Table 7. The diagnostic expectation can generate housing price growth volatility which is twice as much as that of rational expectation. More considerations in forming reference point makes the DE11 generate relatively smoother responses than DE1, while still significantly larger than the case under RE.

Chapter 5

Impact of Diagnostic Expectation in the New Keynesian Model

This chapter displays the main results from model simulation. The shocks will affect the housing price dynamics both through extensive margin and intensive margin as specified by the decomposition derived from first-order conditions. The diagnostic expectations, working through both margins, lead to distinct responses of the real economy from rational expectations. To illustrate this, I first present the impulse response functions (IRF) of key variables to one-standard-deviation shocks in total factor productivity, m_t, and the monetary policy. The results under rational expectation, diagnostic expectation with past one period as the reference point (DE1), and diagnostic expectation with past three years as the reference point (DE11) are shown for comparison. Besides, the separate IRFs for savers and investors reveal that the main driving force of housing price dynamics comes from the housing investors. In the second section, the forecast error variance decomposition (FEVD) results are presented. As indicated by the empirical FEVDs covered in the introduction part, the TFP shock and MP shock contribute significantly to the housing price forecast error variance.

5.1 Housing Price Decomposition

To understand how the diagnostic expectation will affect the decision-making, I analyze the housing price with the following first-order conditions derived from my model: housing demand Equation 4.4, housing distribution Equation 4.5, and optimal borrowing condition 4.6.

Aggregate Housing Price. This analysis refers to how the shocks will affect the household's total housing demand, including residential property and industrial property. The substitution effect (SE) will be negative for housing holding when the relative housing price increase after a positive TFP shock, indicated by the negative relationship of q_t and H_t in Equation 4.4. The income effect (IE) will be positive and come from two channels: production return and investment return. The increase in $\mathbb{E}^{\theta} q_{t+1}$ and $\mathbb{E}^{\theta} r_{h,t+1}$ lead to higher income in the future and relaxed budget constraint. As a result, total housing demand increases and thus the housing price. Note here, the SE directly works within the same period while the IE directly works through affecting the future. The diagnostic expectation will amplify the IE more by generating overreaction through biased expectation. Under both rational expectation and diagnostic expectation, the IE dominates SE, leadind to positive responses in housing demand and housing price. While under the diagnostic expectation, the IE is even larger, and the response in normal good consumption and housing good consumption increase even more. The DE strengthens the extensive margin responses and could help to generate large volatility in housing price dynamics.

Housing Value Distribution. This analysis refers to the distribution of total house holding into residence use and production use, the ϕ parameter specified in the model. From Equation 4.5, the net residential needs are also affected by the relative magnitude of SE and IE. After a positive TFP shock, the expected marginal product of industrial real estate will be larger, and this leads to the relative price of residential housing consumption increase: SE leads to a decrease in residential housing holding ϕ_t. While at the same time, the IE leads to a positive response in ϕ_t. To see this in both Equation 4.5 and Equation 4.4, the increase in $\mathbb{E}^{\theta} q_{t+1}$ and $\mathbb{E}^{\theta} r_{h,t+1}$ leads to increase in ϕ_t keeping other terms constant. Also, the increase in future consumption due to IE leads to an increase in today's residential housing holding by Equation 4.5. Note here, the IE is stronger for investors than for savers due to the extra term $\mathbb{I}_2 m_t \psi_t \mathbb{E}_t^{\theta} q_{t+1}$ since the $m_t \psi_t$ is strictly positive. The diagnostic expectation will affect both the SE and IE since they are all affected by the future belief directly. Later in the IRF part, we will see that the diagnostic expectation strengthens the IE more and can generate the net positive responses in ϕ_t, consistent with the empirical findings.

5.2 Impulse Response Functions

This section explores the impact of diagnostic expectation on housing price and other macroeconomic variables by simulating my model both under DE and RE to a one standard deviation shock in either TFP, m_t, or monetary policy. By comparing the IRFs under different belief-formation processes in magnitude and direction, I reveal the mechanism of overreaction generated by the DE. Under my framework, housing has three roles: residential real estate as a consumption good, industrial real estate as a production input, and total housing as an investment good. The savers' and investors' housing holding responses are presented from these three motivations. In the end, differences in responses under the same expectation formation process with different monetary policy rules are presented, indicating the importance of policy guidance I want to focus more on in the future.

To make the results comparable, Figure 2 shows the empirical responses of real housing prices following local projection method proposed by Jordà [19].

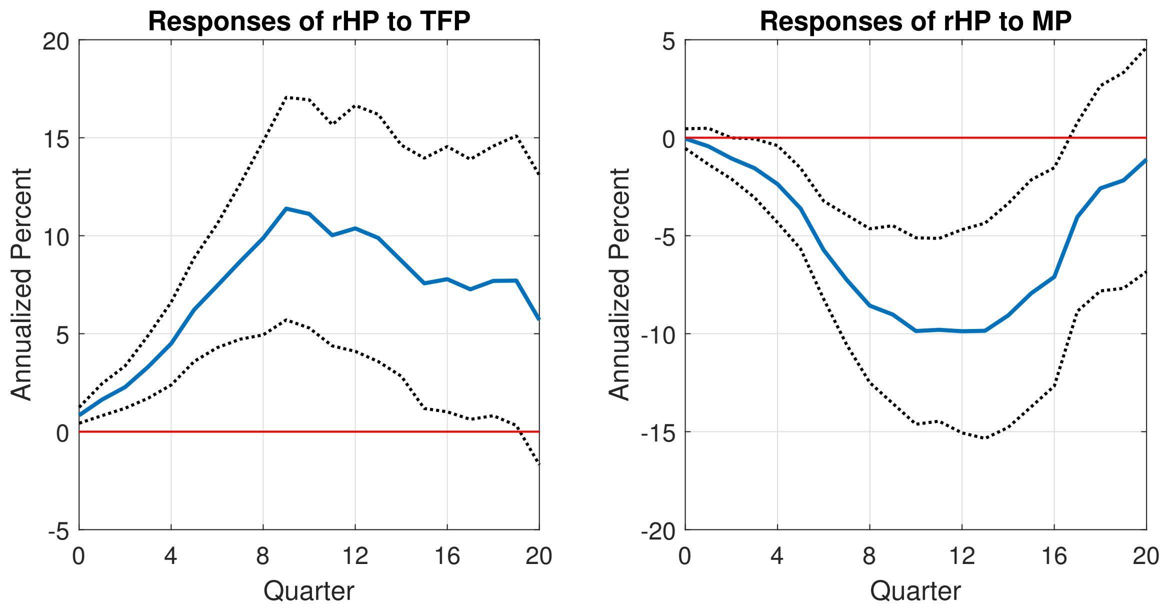


Figure 2: Local projection, Real Housing Price

Notes: The real housing price data is constructed from Freddie Mac housing price index inflated with the CPI less shelter. The TFP shocks are identified by Fernald [15] and the MP shocks are identified by Romer and Romer [29] and extended by Coibion et al. [11]. The standard deviation of these three shocks are 3.1857 and 0.5741. The graph display the empirical impulse responses with 90% bootstrap confidence interval. The sample covers 1975Q1 - 2008Q4. The unit of y-axis is annualized percentage responses with the unit of x-axis denoting for quarters.

IRFs to TFP shock

Magnitude. The first impact of DE is reflected in the difference in the magnitude of responses to the same TFP shock. Under the diagnostic expectation, over-optimism after the positive TFP shock leads the agent to consume twice the amount under rational expectation. This large jump of consumption comes from the consumption smoothing motivation resulting from a perceived

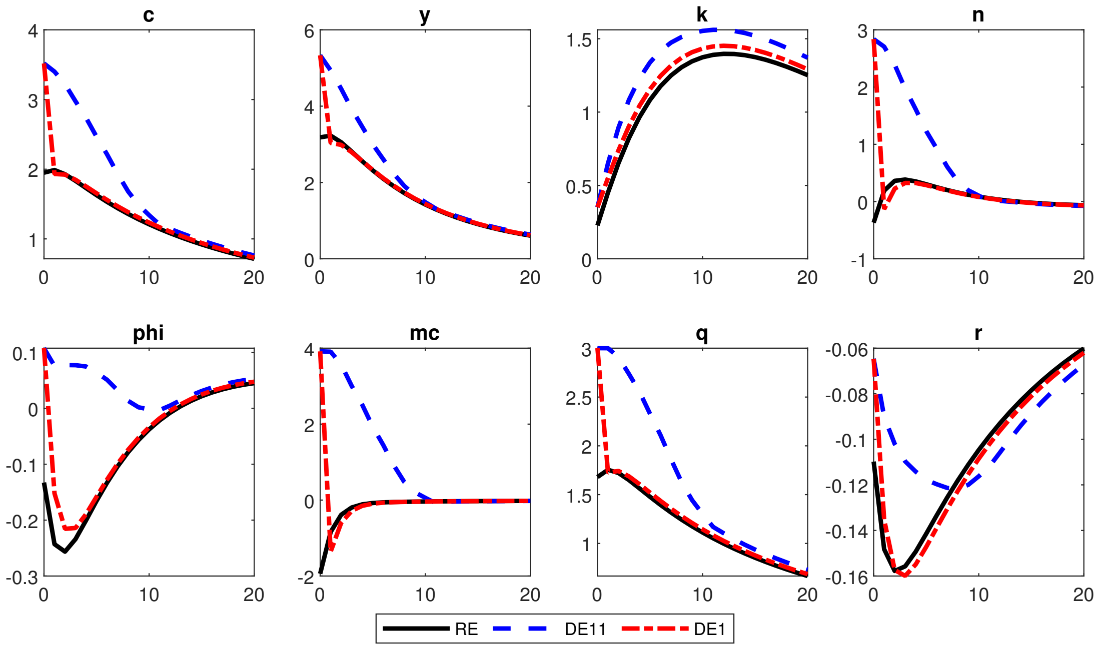


Figure 3: Impulse Response Functions, TFP shock, Taylor Rule 1

Notes: This figure displays the annualized percentage response to 1 std shock in TFP under Talyor rule 1.

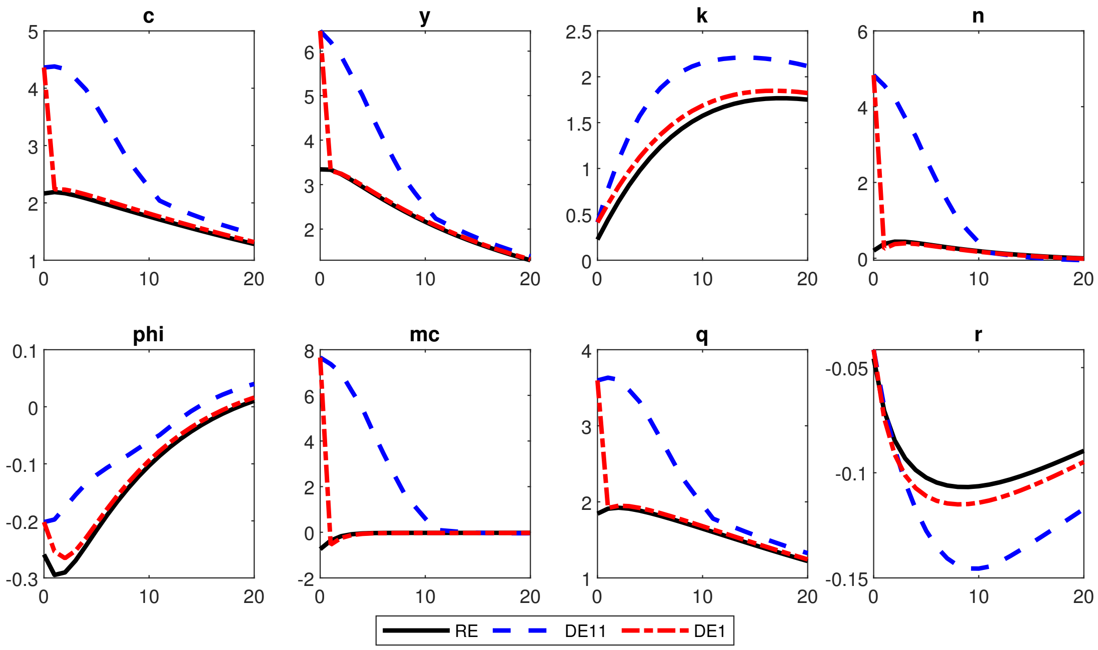


Figure 4: Impulse Response Functions, TFP shock, Taylor Rule 2

Notes: This figure displays the annualized percentage response to 1 std shock in TFP under Talyor rule 2.

wealth effect. The expected wage increase also leads to an overreaction in labor supply, boosting production and increasing the marginal cost. Also, The diagnostic expectation helps generate a counter-cyclical markup that cannot be realized under rational expectations. The housing price responded by 3.6% under DE while only 1.8% under RE. Refer to the local projection results in figure 2, the rational expectation in the standard NK framework cannot generate enough housing price response in magnitude.

Persistency. Between two versions of diagnostic expectation different in reference point, the diagnostic expectation with 3-year memory recall can generate more persistent responses in consumption, residential housing demand, marginal cost, and housing price. The short-memory recall DE displays overreaction in the first place and quickly converges back to the case of rational expectation since the one-time shock here generates no surprises later and thus no extra consumption smoothing and responses relative to rational expectation.

Residential Housing Share. Another interesting pattern generated by DE system under the first Taylor rule is the positive response of residential housing value share (ϕ in the model). Under the rational expectation, the positive TFP shock leads to a decrease in the proportion of housing allocated for residential use and, therefore, increased commercial use since the marginal production of industrial real estate input increases. In comparison, the DE agents' overoptimism will increase their residential housing good consumption due to the dominated income effect. The increase in residential housing value relative to output is consistent with the empirical results from the local projection of residential value share to TFP shock (Figure 5). Although the increase is not quantitatively enough, the rational expectation NK model cannot generate this pattern. Such a pattern disappears after monetary authority considers future expected inflation and the output gap in Taylor rule 2.

To understand the mechanism of this positive response in residential housing consumption, I plot the IRFs of savers and investors in Figure 6 and Figure 7 under Taylor rule 1 and Taylor rule 2 separately. The diagnostic expected next period inflation and output gap showing up in Taylor rule 2 lead to a further decrease of nominal interest rate, inducing a lower return in lending out money for the saver and a smaller proportion of residential housing consumption. The holding of housing assets for the investor will increase since they are willing to borrow to invest in the housing market, and their optimistic expectation on next period housing price give the extra motivation. The housing price increases due to the relative demand jump from investors and the house ownership transfer to investors from savers with constant supply. In re-

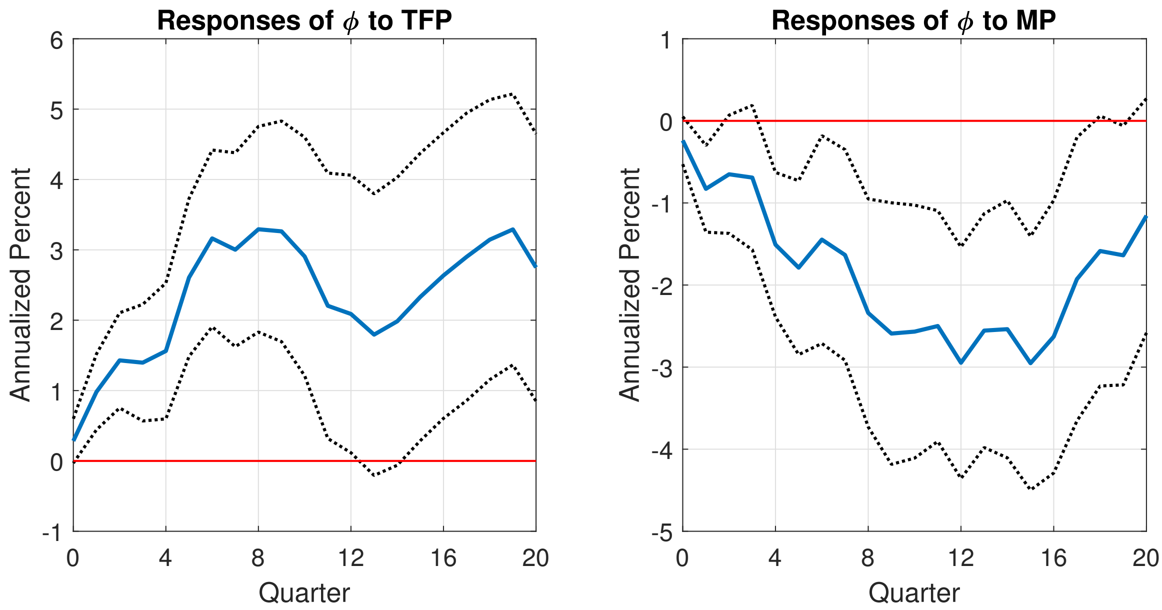


Figure 5: Local projection, Residential Housing Value Share

Notes: The sample spans from 1971Q1 to 2007Q4 with TRF shock form John Fernald's website, with standard deviation 3.1857. The response are annualized percentage response to 1 std shock. The residential housing value share is calculate by dividing owner-occupied real estate at market value level data with gross domestic product data. The owner-occupied real estate market value data is from Flow of Fund Table B.101.

sponse to the increase in industrial real estate return, the net effect of saver is negative while the net effect of investors is positive. The relative magnitude of the increase determines the total net effect of the residential housing value for the economy in industrial real estate return for savers; investors are always over-optimistic due to extra holding on housing assets and over-optimism from DE. Considering the case under the second Taylor rule, the responses in ϕ are negative due to the sharp increase in return of industrial real estate, and the relative price of residential housing consumption is higher than the case under Taylor rule 1. Savers reduce the residential housing consumption more than under Taylor rule 1, and the investor does not increase enough.

Another setting worth mentioning is the weight in constructing the reference point matters for the shape of impulse response functions. The shape displayed in Figure 3 relies on a weighting vector that concentrate more on the medium-length memory. This could be interpreted as the individual has rigidity in updating reference points and tend to have more confidence in their past expectations and less on those that are latest or the far from now.

IRFs to m_t and MP shock

As shown in Figure 8 and Figure fig:IRFmp1MP, the IRFs except that of housing price under DE and RE do not vary much to both shocks in m_t and monetary policy. This holds under

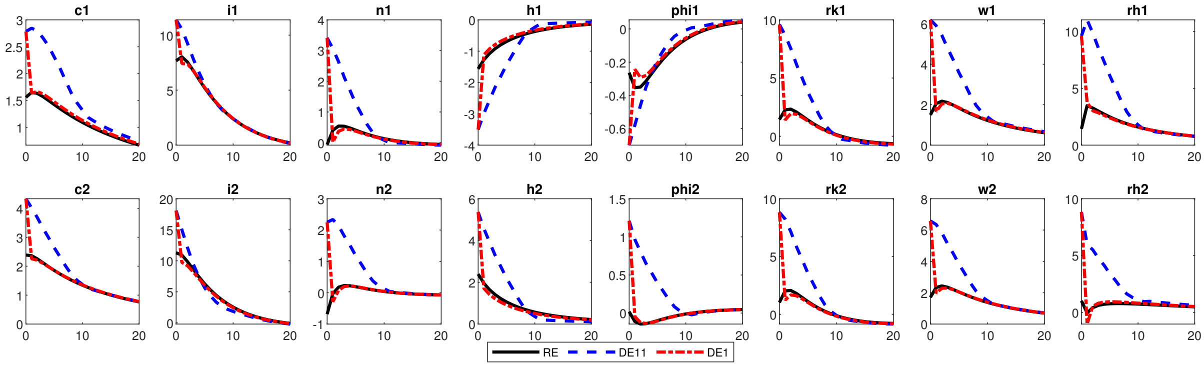


Figure 6: Separate Impulse Response Functions, TFP shock, Taylor Rule 1

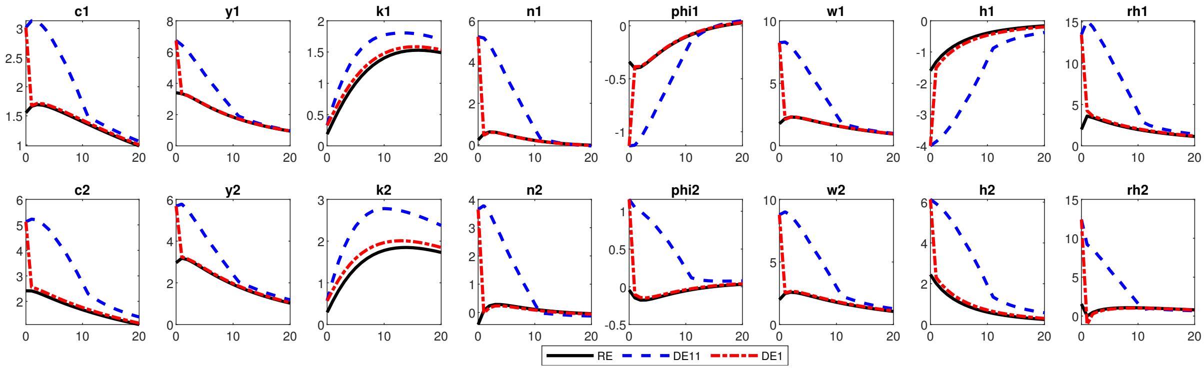


Figure 7: Separate Impulse Response Functions, TFP shock, Taylor Rule 2

both Taylor rule 1 and Taylor rule 2. Furthermore, the model cannot generate the negative empirical response in residential housing share to monetary policy shock. The reason behind these observations is that the transition mechanism induced by the m_t shock and monetary policy shock is different from that induced by the TFP shock.

Both the m_t shock and MP shock firstly affect the borrowing constraint either through adjusting the tightness of borrowing constraint or influencing the interest rate. With sticky prices, the real interest rate will be affected, and thus the consumption and output. The decreasing housing price will amplify the discouragement in real economy. But without habit-formation and other rigidities in my model, these effects do not have much persistency. At the same time, these two shocks do not directly affect the real economy, while the TFP shock affects the production and returns in the first place. Under the IE and SE analysis framework as I have before, the IE's effect is much smaller in the case with m_t and MP shocks due to both lower persistency and indirect channel. The DE mainly plays the role through the income channel and consumption smoothing, and here it has a limited effect, leading to the IRFs not have many differences.

As for the residential housing share, the m_t shock and MP shock lead to a decrease in

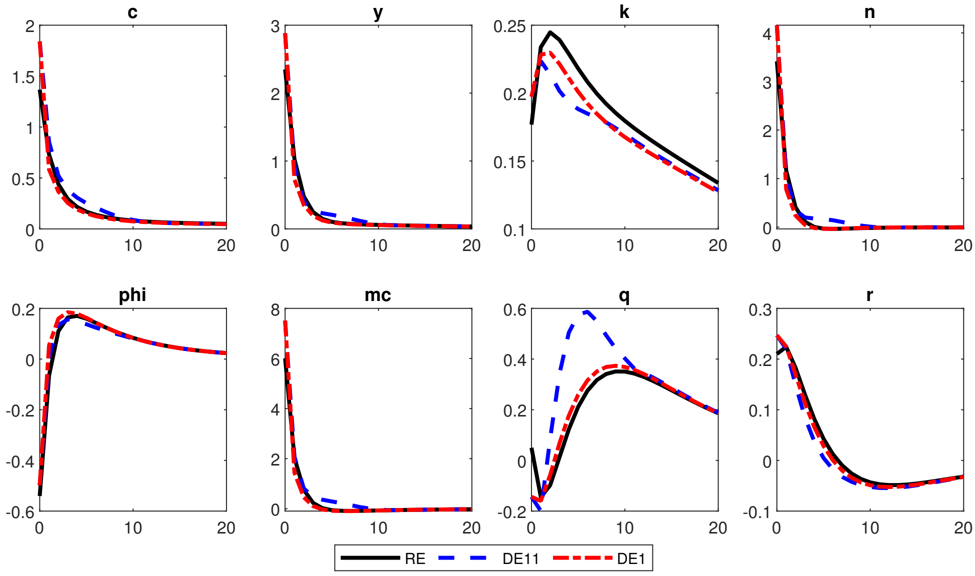


Figure 8: Impulse Response Functions, m_t Shock, Taylor Rule 1

Notes: This figure displays the annualized percentage response to 1 std shock in m_t under RE, DE1 and DE11

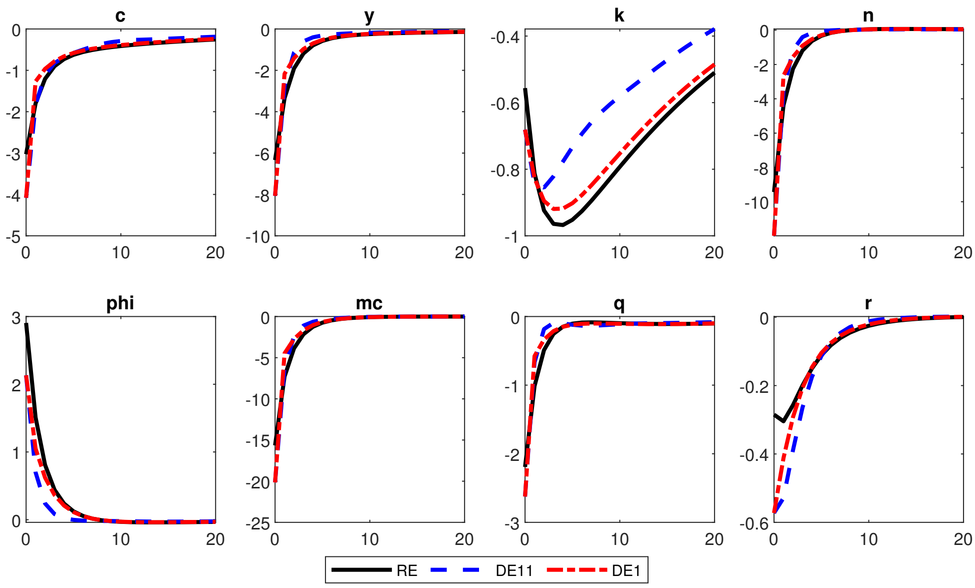


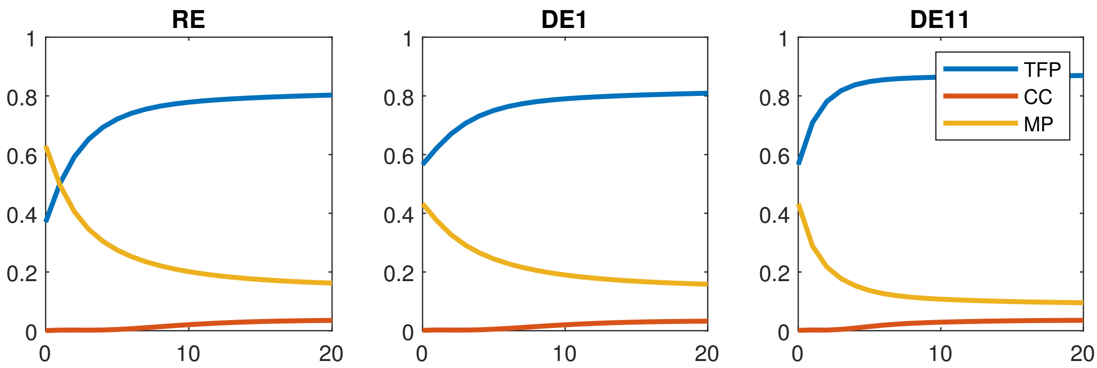
Figure 9: Impulse Response Functions, Monetary Policy Shock, Taylor Rule 1

Notes: This figure displays the annualized percentage response to 1 std shock in monetary policy under RE, DE1 and DE11 industrial real estate; therefore, it should be accompanied by a decrease in industrial using share and an increase in residential using share according to the substitution effect. In contrast, the income effect indicates reducing consumption in residential housing. Although the total housing demand decrease, the relative residential housing share increase when SE dominates IE, which is the case shown by Figure 8 and Figure 9. The weaker IE comes from the weaker DE for the reasons discussed before.

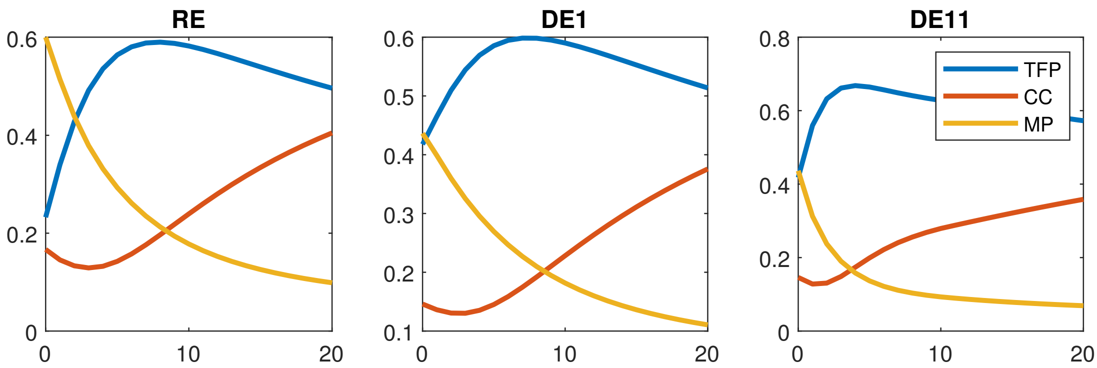
5.3 Forecast Error Variance Decomposition

The following Figure 10 displays the forecast error variance decomposition results generated from the model. The main finding is that the DE amplifies the effect of TFP shock and weakens the effect of monetary policy shock on housing price. The relatively more minor contribution of monetary policy shock is closer to the empirical FEVDs as shown before (Figure 1). The contributing power of m_t shock is not affected significantly by the magnitude of DE and the reference point of DE. This finding holds under both Taylor rule 1 and 2.

Another point worth mentioning is the contribution of m_t shock, which I allocate all the unspecified shock into this shock. Under the Taylor rule 2, the m_t shock contributes to the FEVD of housing price by 20% to 40 % at maximum under diagnostic expectation and 60% at maximum under rational expectation. The empirical implied should be around 60%. However, due to the relatively simple model setting here, this needs further discussion in middle-scale macroeconomic models.



(a) Housing Price,Taylor Rule 1



(b) Housing Price, Taylor Rule 2

Figure 10: FEVDs, Real Housing Price

Chapter 6

Conclusion

In this paper, I test the predictability of forecast errors in the housing market and find evidence for deviation of rational expectation. Incorporating the diagnostic expectation in a two-agent New Keynesian framework with the approach proposed by Bianchi et al. [4], this paper finds the existence of representative heuristic leads to more persistence and more significant responses in housing price to TFP shock. The increasing residential housing value share to positive TFP shock can also be resolved under the DE. Overestimation, especially from investors, leads to an overreaction in consumption, investment, and housing demand.

Based on the current findings in this paper, I propose to dig deeper in several directions: firstly, what will be the case when the diagnostic expectation is only restricted to a proportion of participants in the economy: for example, the housing investors; secondly, whether the central bank's expectation is affected by the representative heuristic; thirdly, what should be the policy implication under diagnostic expectation circumstances.

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Chapter 7

Appendix

7.1 Detailed Derivation of Two-agent New Keynesian Model

7.1.1 Households: Saver and Investor

$$\max_{C_{it},N_{it},K_{it},H_{it},\phi_{it},b_t} \mathbb{E}_0^{\theta} \sum_{t=0}^{\infty} \beta_i^t [lnC_{it} + Jln(\phi_{it}H_{it}) - \chi \frac{N_{it}^{1+\eta}}{1+\eta}]$$

subject to the budget constraint:

$$\begin{aligned} s.t.\ & C_{it} + K_{it} + q_t(H_{it} - H_{i,t-1}) + (\mathbb{I}_1 - \mathbb{I}_2)b_t + \frac{\varphi_k}{2}(\frac{I_{it}}{K_{i,t-1}} - \delta)^2 K_{i,t-1} \\ & = r_{i,k,t}K_{i,t-1} + r_{i,h,t}(1-\phi_{i,t-1})H_{i,t-1} + w_{it}N_{it} + (1-\delta)K_{i,t-1} + \Pi_t + (\mathbb{I}_1 - \mathbb{I}_2)\frac{R_{t-1}}{1+\pi_t}b_{t-1} \end{aligned} \tag{7.1}$$

subject to the credit constraint:

$$b_t \leq m_t \mathbb{E}_t(q_{t+1}\frac{H_{2t}}{R_t}) \tag{7.2}$$

Set down the lagrangian:

$$\begin{aligned} \mathscr{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \{ & lnC_{it} + jln(\phi_{it}H_{it}) - \chi \frac{N_{it}^{1+\eta}}{1+\eta} \\ & + \lambda_t[r_{i,k,t}K_{i,t-1} + r_{i,h,t}(1-\phi_{i,t-1})H_{i,t-1} + w_{it}N_{it} + (1-\delta)K_{i,t-1} + \Pi_t + (\mathbb{I}_1 - \mathbb{I}_2)\frac{R_{t-1}}{1+\pi_t}b_{t-1} \\ & - C_{it} - K_{it} - q_t(H_{it} - H_{i,t-1}) - (\mathbb{I}_1 - \mathbb{I}_2)b_t - \frac{\varphi_k}{2}(\frac{I_{it}}{K_{i,t-1}} + \delta)^2 K_{i,t-1}]\} \\ & + \mathbb{I}_2 \psi_t[m_t \mathbb{E}_t(q_{t+1}H_{2t}) - b_t R_t] \end{aligned}$$

Taking FOCs with respect to C_{it}, K_{it}, N_{it}, H_{it}, ϕ_{it}, b_t:

$$C_{it}^{-1}[1+\varphi_k(\frac{I_{it}}{K_{i,t-1}}-\delta)] = \beta_i \mathbb{E}_t\{C_{i,t+1}^{-1}[r_{i,k,t+1}+(1-\delta)-\frac{\varphi_k}{2}(\frac{I_{i,t+1}}{K_{it}}-\delta)^2+\varphi_k(\frac{I_{i,t+1}}{K_{it}}-\delta)(\frac{I_{i,t+1}}{K_{it}}+1-\delta)]\} \tag{7.3}$$

$$\chi N_{it}^{\eta} = C_{it}^{-1} w_{it} \tag{7.4}$$

$$jH_{it}^{-1} + \beta \mathbb{E}_t[C_{i,t+1}^{-1}(r_{i,h,t+1}(1-\phi_{it})+q_{t+1})] + \mathbb{I}_2 m_t \psi_t \mathbb{E}_t q_{t+1} = C_{it}^{-1} q_t \tag{7.5}$$

$$j\phi_{it}^{-1} = \beta_i \mathbb{E}_t[C_{i,t+1}^{-1} r_{i,h,t+1} H_{it}] \tag{7.6}$$

$$C_{it}^{-1} = \beta_i \mathbb{E}_t[C_{i,t+1}^{-1}\frac{R_t}{1+\pi_{t+1}}] + \mathbb{I}_2 \psi_t R_t \tag{7.7}$$

7.1.2 Firms

$$P_t = (\omega P_{t-1}^{1-\theta} + (1-\omega)P^{*1-\theta})^{\frac{1}{1-\theta}} \tag{7.8}$$

$$w_{it} = (1-\alpha-\kappa) mc_t \frac{Y_{i,j,t}}{N_{i,j,t}} \tag{7.9}$$

$$r_{i,k,t} = \alpha mc_t \frac{Y_{i,j,t}}{K_{i,j,t-1}} \tag{7.10}$$

$$r_{i,h,t} = \kappa mc_t \frac{Y_{i,j,t}}{(1-\phi_{i,t-1})H_{i,j,t-1}} \tag{7.11}$$

The marginal cost can be solved with the above FOCs and production function:

$$mc_t = \frac{1}{A_t}(\frac{r_{k,t}}{\alpha})^{\alpha}(\frac{r_{h,t}}{\kappa})^{\kappa}(\frac{w_t}{1-\alpha-\kappa})^{1-\alpha-\kappa} \tag{7.12}$$

$$\frac{p_{j,t}^*}{P_t} = \frac{\theta}{\theta-1}\frac{\mathbb{E}_t[\sum_{i=0}^{\infty}\omega^i\beta^i mc_{t+i}(\frac{P_{t+i}}{P_t})^{\theta}]}{\mathbb{E}_t[\sum_{i=0}^{\infty}\omega^i\beta^i(\frac{P_{t+i}}{P_t})^{\theta-1}]} \tag{7.13}$$

7.1.3 Monetary Authority

The central bank follows a nominal interest rule in response to inflation and output gap deviation from its steady state value, here we consider the relatively simple version Taylor Rule 1 and the complete version Taylor Rule 2:

$$\frac{R_t}{R} = (\frac{R_{t-1}}{R})^{a_1}[(\frac{\pi_t}{\pi})^{a_2}]^{1-a_1}e^{\nu_t} \tag{7.14}$$

$$\frac{R_t}{R} = (\frac{R_{t-1}}{R})^{a_1}[(\frac{\mathbb{E}_t^{\theta}(\pi_{t+1})}{\pi})^{a_2}X_t^{a_3}]^{1-a_1}e^{\nu_t} \tag{7.15}$$

7.1.4 Equilibrium

With the household budget constraint (7.1) and government budget constraint (**??**) and the zero profit from assuming competitive final good market, we have the consumption good market clear:

$$C_{1t} + I_{1t} + C_{2t} + I_{2t} = \frac{1}{g}Y_t \tag{7.16}$$

And the budget constraint of two type of agents in the economy:

$$C_{1t} + I_{1t} + \frac{\varphi_k}{2}(\frac{I_{1t}}{K_{1,t-1}} - \delta)^2 K_{1,t-1} + q_t(H_{1t} - H_{1,t-1}) + b_t = \frac{1}{g_t}Y_{1t} + \frac{R_t}{1+\pi_{t-1}}b_{t-1} \tag{7.17}$$

$$C_{2t} + I_{2t} + \frac{\varphi_k}{2}(\frac{I_{2t}}{K_{2,t-1}} - \delta)^2 K_{2,t-1} + q_t(H_{2t} - H_{2,t-1}) + \frac{R_t}{1+\pi_{t-1}}b_{t-1} = \frac{1}{g_t}Y_{2t} + b_t \tag{7.18}$$

The housing market clearing with total housing equals to residential and commercial housing demand:

$$\phi_{it}H_{it} + (1-\phi_{it})H_{it} = H_{it} \tag{7.19}$$

With law of motion of capital:

$$K_{it} = I_{it} + (1-\delta)K_{i,t-1} \tag{7.20}$$

Binding credit constraint:

$$b_t R_t = m_t \mathbb{E}_t q_{t+1} H_{2t} \tag{7.21}$$

Steady State

At steady state, we have $A_t = 1, H_t = H, \pi_t = 0$ To solve the steady state, we need to first assume d denote the proportion of housing owned by the patient households and then solve the d and q jointly from the housing market clearing condition.

$$R = \frac{1}{\beta_1}$$

$$r_{1k} = \frac{1}{\beta_1} - (1-\delta) \quad r_{2k} = \frac{1}{\beta_2} - (1-\delta)$$

$$mc = \frac{\theta - 1}{\theta}$$

$$r_{1k} = (1-\alpha-\kappa)mc\frac{Y_1}{K_1} \quad r_{2k} = (1-\alpha-\kappa)mc\frac{Y_2}{K_2}$$

$$\frac{Y_1}{K_1} = \frac{r_{1k}}{(1-\alpha-\kappa)mc} \quad \frac{Y_2}{K_2} = \frac{r_{2k}}{(1-\alpha-\kappa)mc}$$

$$C_1 + \delta K_1 = \frac{1}{g}Y_1 \quad C_2 + \delta K_2 = \frac{1}{g}Y_2$$

$$\frac{C_1}{Y_1} = \frac{1}{g} - \delta\frac{K_1}{Y_1} \quad \frac{C_2}{Y_2} = \frac{1}{g} - \delta\frac{K_2}{Y_2}$$

$$\frac{C_1}{K_1} = \frac{1}{g}\frac{Y_1}{K_1} - \delta \quad \frac{C_2}{K_2} = \frac{1}{g}\frac{Y_2}{K_2} - \delta$$

$$\xi N_1^{\eta+1} = (1-\alpha-\kappa)mc\frac{Y_1}{C_1} \quad \xi N_2^{\eta+1} = (1-\alpha-\kappa)mc\frac{Y_2}{C_2}$$

$$\frac{K_1}{N_1} = [\frac{Y_1}{K_1}N_1\kappa((1-\phi_1)dH)^{-\kappa}]^{\frac{1}{\alpha-1}} \quad \frac{K_2}{N_2} = [\frac{Y_2}{K_2}N_2\kappa((1-\phi_2)(1-d)H)^{-\kappa}]^{\frac{1}{\alpha-1}}$$

$$\phi_1 = \frac{j}{j + \beta_1\frac{Y_1}{C_1}\kappa mc} \quad \phi_2 = \frac{j}{j + \beta_2\frac{Y_2}{C_2}\kappa mc}$$

$$j\frac{C_1}{dH} + \beta_1[\kappa mc\frac{Y_1}{dH} + q] = q = j\frac{C_2}{(1-d)H} + \beta_2[\kappa mc\frac{Y_2}{(1-d)H} + q] + m\psi q$$

$$b = mtq\frac{H_2}{R}$$

$$\psi = \frac{\beta_1 - \beta_2}{C_2}$$

Log-linearization

Goods market clearing for two agents:

$$C_1 c_{1t} + I_1 i_{1t} + C_2 c_{2t} + I_2 i_{2t} = \frac{Y_1}{g} y_{1t} + \frac{Y_2}{g} y_{2t} \tag{7.22}$$

Law of motion of capital

$$k_{i,t} = \delta i_{i,t} + (1-\delta) k_{i,t-1} \tag{7.23}$$

Saver's budget constraint:

$$C_1 c_{1t} + I_1 i_{1t} + qH_1(h_{1t} - h_{1,t-1}) + bb_t = rb(r_{t-1} - \pi_t + b_{t-1}) + \frac{Y_1}{g} y_{1t} \tag{7.24}$$

Investors' budget constraint:

$$C_2 c_{2t} + I_2 i_{2t} + qH_2(h_{2t} - h_{2,t-1}) + rb(r_{t-1} - \pi_t + b_{t-1}) = \frac{Y_2}{g} y_{2t} + bb_t \tag{7.25}$$

Housing demand for saver:

$$-jH_1^{-1} h_{1t} + \frac{\beta_1}{C_1} \mathbb{E}\{-[r_{1,h}(1-\phi_1) + q] c_{1,t+1} - r_{1,h}\phi_1 \phi_{1t} + r_{1,h}(1-\phi_1) r_{1,h,t+1} + q q_{t+1}\} = -\frac{q}{C_1} c_{1t} + \frac{q}{C_1} q_t \tag{7.26}$$

Housing demand for investor

$$-jH_2^{-1} h_{2t} + \frac{\beta_2}{C_2} \mathbb{E}\{-[r_{2,h}(1-\phi_2) + q] c_{2,t+1} - r_{2,h}\phi_2 \phi_{2t} + r_{2,h}(1-\phi_2) r_{2,h,t+1} + q q_{t+1}\} + \psi m q \mathbb{E}[m_t + \psi_t + q_{t+1}] = \tag{7.27}$$

Euler equation on capital:

$$-c_{i,t} + \varphi_k \frac{I_i}{K_i}(i_{i,t} - k_{i,t-1}) = -c_{i,t+1} + \beta_i \mathbb{E}_t[r_{i,k} r_{i,k,t+1} + \varphi_k \frac{I_i}{K_i}(i_{i,t+1} - k_{i,t})] \tag{7.28}$$

Euler equation on labor:

$$\eta n_{i,t} = -c_{i,t} + w_{i,t} \tag{7.29}$$

Euler equation on share of residential housing:

$$-\phi_{i,t} = -c_{i,t+1} + r_{i,h,t+1} + h_{i,t} \tag{7.30}$$

Saver's decision on saving:

$$-c_{1t} = -c_{1,t+1} + r_t - \pi_{1,t+1} \tag{7.31}$$

Investor's decision on lending money:

$$-\frac{1}{C_2}c_{2t} - \psi R\psi_t - \psi r_t = \frac{\beta_2}{C_2}\frac{R}{1+\pi}[-c_{2,t+1} + r_t - \pi_{t+1}] \tag{7.32}$$

Binding credit constraint:

$$b_t = m_t + q_{t+1} + h_{2t} - r_t \tag{7.33}$$

Production function:

$$y_{i,t} = a_t + \alpha k_{i,t-1} + \kappa h_{i,t-1} + (1-\alpha-\kappa)n_{i,t} - \kappa\frac{\phi_i}{1-\phi_i}\phi_{i,t-1} \tag{7.34}$$

Labor demand:

$$w_{it} = mc_t + y_{it} - n_{it} \tag{7.35}$$

Capital demand:

$$r_{i,k,t} = mc_t + y_{it} - k_{i,t-1} \tag{7.36}$$

Commercial housing demand:

$$r_{i,h,t} = mc_t + y_{it} - h_{i,t-1} + \frac{\phi_i}{1-\phi_i}\phi_{i,t-1} \tag{7.37}$$

Total supply: NKPC

$$\pi_t = \beta_1 \mathbb{E}_t \pi_{t+1} + \frac{(1-\omega\beta)(1-\omega)}{\omega}mc_t \tag{7.38}$$

Taylor rule:

$$r_t = \psi_1 r_{t-1} + (1-\psi_1)\psi_2\pi_t + \nu_t \tag{7.39}$$

Fisher equation, rr_t denote for real interest rate,

$$rr_t = r_t - \pi_t \tag{7.40}$$

TFP shocks:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{7.41}$$

Liquidity constraint shock:

$$m_t = \rho_m m_{t-1} + \varepsilon_{m,t} \tag{7.42}$$

Monetary policy shocks:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t} \tag{7.43}$$